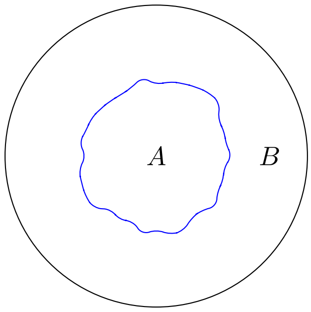
TAMS42 (Probability and Statistics) Vinjett 4
—Monte Carlo method

Background: An area that can not be computed directly maybe estimated approximately using numerical methods. One of these methods is the *Monte Carlo method* which uses an area-computable region covering an area-uncomputable region, and makes an approximation by using random points. Based on the following graph, let us review a particular Monte Carlo method: the hit-or-miss method.



In $\mathbb{R}^2$, let A be an area-uncomputable region and B be an area-computable region such that A is wholly contained in B. Random points are generated **uniformly** within the region B, hence a point lands in the region A with probability $|A|/|B|$, where $|A|$ and $|B|$ are areas of regions A and B. Suppose we have n such random points. How to estimate the area $|A|$ of the region A based on these n random points? If we observe that m such points have landed in the region A, then the hit-or-miss method tells us that the area $|A|$ can be estimated as

$$|A| \approx \frac{m}{n} \cdot |B|.$$

For instance, if $|B| = 1, n = 100$ and $m = 10$, then $|A| \approx 0.1$.

Discussions: (a) In order to have a better estimation of $|A|$, do we increase or decrease the number n of random points?

(b) Why the random points are generated **uniformly**? Can they be Normal (or other) random variables?

(c) For a fixed n, which choice of B will likely provide a better estimation of $|A|$?

(i) B is area-computable and is much larger than A;

(ii) B is area-computable and is about twice bigger than A;

(iii) B is area-computable and is very close to A.

(d) Can you think of other applications of the hit-or-miss method? (For example, in $\mathbb{R}^3$, one can similarly use the method to estimate the volume of a volume-uncomputable object).