Lecture 1

$$c_{ij} = \sum_{k=1}^{n} a_{ik} b_{kj}$$

(1)
p. 6

Load $B(:,j)$, $C(:,j)$

for $k = 1:n$
  Load $A(:,k)$
  $C(:,j) = C(:,j) + A(:,k) * B(k,j)$

Total $O(n^3)$ multiplications / memory access

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(2)
p. 8

Change from column to block storage requires $n \times p = n\sqrt{n} = n^{1.5}$ memory load / store

The algorithm is

Load $C_{ij} \in \mathbb{R}^{p \times p}$

for $k = 1:p$
  Load $A_{ik}, B_{kj} \in \mathbb{R}^{p \times p}$
  $C_{ij} = C_{ij} + A_{ik} \cdot B_{kj}$ $O(p^3)$ mults.

Block size is $p = \sqrt{n}$ so work

$$p^2 \cdot p \cdot p^3 = p^6 = (\sqrt{n})^6 = n^3 \text{ mult.}$$

and need

$$p^2 \cdot p \cdot 2 = 2p^3 = 2n^{1.5} \text{ memory loads}$$

(3)

p.12

$$y = \underbrace{\underbrace{(AB)}_{n^3} x}_{n^2} = \underbrace{A \underbrace{(Bx)}_{n^2}}_{n^2}$$

Either $n^3 + n^2$ or $2n^2$ multiplications.

Matlab evaluates from left to right

```
>> n=10^3; A=rand(n,n); B=rand(n,n);
>> x=rand(n,1)
>> tic, y=A*B*x; toc   => 4.4 s
>> tic, y=A*(B*x); toc => 0.002 s
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(4)

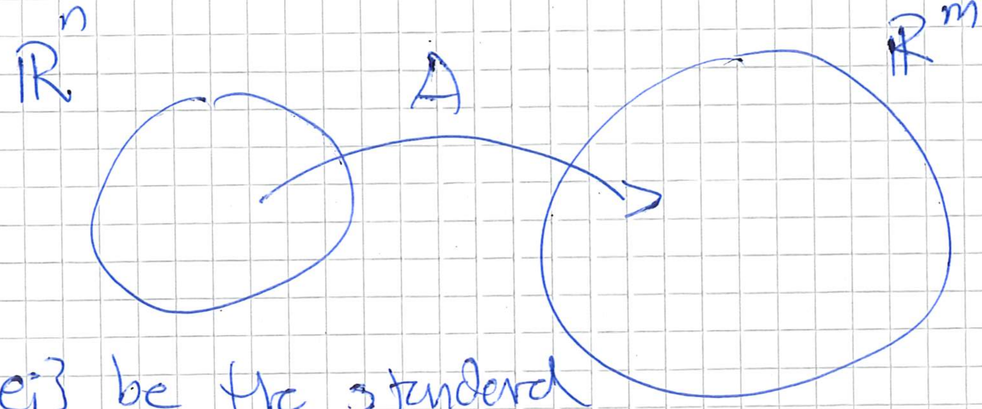
p.13

$A_{ij} = u_i \cdot v_j \quad 1 \le i,j \le n \Rightarrow n^2$ multiply. scalar.

Example $\quad y = (uv^T)x = \underbrace{u \cdot \underbrace{(v^T x)}_{n \text{ op}}}_{n \text{ op}}$

Need $O(n)$ multiplications and memory.

5
P.16



Let $\{e_i\}$ be the standard basis and $A=(a_1, a_2, .., a_n)$, $a_k \in \mathbb{R}^m$

Then

$$Ae_i = a_i \in \text{Range}(A) \text{ for all } i.$$

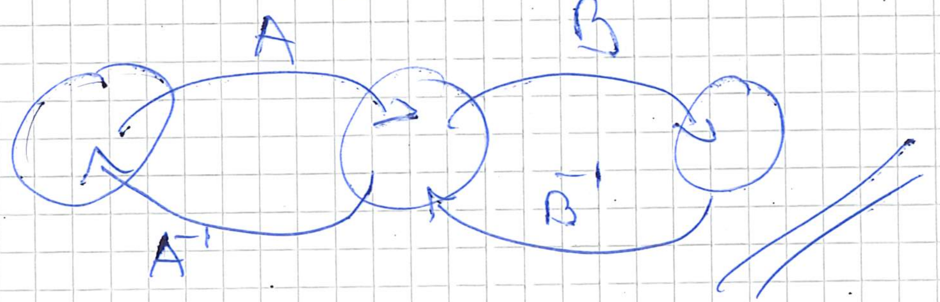
So

$$\text{Range}(A) = \text{span}(a_1, .., a_n) \subset \mathbb{R}^m$$

6
P.17

$$I = (AB)^{-1}(AB) = B^{-1}A^{-1} \cdot AB = B^{-1}(A^{-1}A)B$$
$$= B^{-1}B = I //$$

or



//

7
P.18

$A \in \mathbb{R}^{m\times n}$. If $b \in \text{Range}(A)$ there is an $x \in \mathbb{R}^n$ so $Ax = b$. This is existance.

If $\dim(\text{Range}(A)) = m$ then $b \in \mathbb{R}^m \equiv \text{Range}(A)$

so existance for all $b \in \mathbb{R}^m$ //

(8) If $Ax=b$ has a solution x_1 and $x_2 \in Null(A)$

P.18

then $A(x_1+x_2) = \underbrace{Ax_1}_{=b} + \underbrace{Ax_2}_{=0} = b$

The solution is unique if $null(A) = \{0\}$

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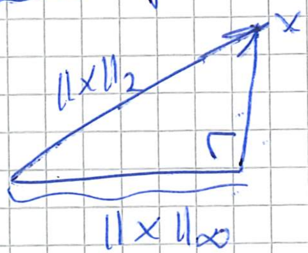
(9) Example $\|x\|_1 = \sum_{i=1}^{n} |x_i|$

P.19



$\|x\|_\infty \leq \|x\|_2 \leq \|x\|_1$

Many different norms and relations //

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(10) $\|A\|_2 = \max_{1\leq i\leq n} \sqrt{\lambda_i(A^TA)}$

P.20

Demanding to compute

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(11) $\|AB\| \leq \max_{x\neq 0} \frac{\|ABx\|}{\|x\|} = \max_{x\neq 0} \frac{\|A(Bx)\|}{\|\underbrace{Bx}_{=y}\|} \cdot \underbrace{\frac{\|Bx\|}{\|x\|}}_{\leq \|B\|}$

P.22

$\leq \|B\| \max_{y\neq 0} \frac{\|Ay\|}{\|y\|} = \|B\| \cdot \|A\|$ //