<u>Def</u> $(V, +, \cdot, \bar{0})$ vektorrum

om

1) V mängd vektorer

2) $+$ addition av vektorer

3) $\cdot$ skalning

4) $\bar{0}$ nollvektor

S.a. räkneregler

- $\bar{u} + \bar{v} = \bar{v} + \bar{u}$
- $\bar{u} + (\bar{v} + \bar{w}) = (\bar{u} + \bar{v}) + \bar{w}$
- $\bar{u} + \bar{0} = \bar{u}$

- till $\bar{u}$ finns (unikt) $-\bar{u}$

  s.a $\bar{u} + (-\bar{u}) = \bar{0}$

- $1 \cdot \bar{u} = \bar{u}$
- $c(\bar{u}+\bar{v}) = c\bar{u} + c\bar{v}$
- $(c+d)\bar{u} = c\bar{u} + d\bar{u}$
- $c(d\bar{u}) = (cd)\bar{u}$

Ex 1 Geom vektorer i planet el rummet. G^2

Ex 2 $Mat(m,n)$

$$\bar{0} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

Ex 3 $\mathbb{R}^n \neq Mat(1,n)$

radvektorer

$$\mathbb{R}^2 = \{(x,y) : x,y \in \mathbb{R}\}$$

$$\bar{0} = (0,0)$$

$(x,y)+(u,v)=(x+u,y+v)$

$5(x,y)=(5x,5y)$

~~**Ex 4** $\mathbb{P} = \{p(t) : p \text{ pol}\}$~~

~~addition: $(1-3t+5t^2)+(4-7t^6)$~~

~~$= 5-3t+5t^2-7t^6$~~

~~nullvektor: nullpolynom 0~~

~~skalning: $5(1-t^2) = 5-5t^2$~~

senare

Kursboken:

$V = \mathbb{R}^4$ (säg)

$\bar{e}_1 = (1,0,0,0)$ $e_2 = (0,1,0,0)$

$\bar{e}_3, \bar{e}_4$

$\mathbb{R}^4 \ni (x_1, x_2, x_3, x_4) =$

$x_1 \bar{e}_1 + x_2 \bar{e}_2 + x_3 \bar{e}_3 + x_4 \bar{e}_4$

$$= (\bar{e}_1 \; \bar{e}_2 \; \bar{e}_3 \; \bar{e}_4) \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix}$$

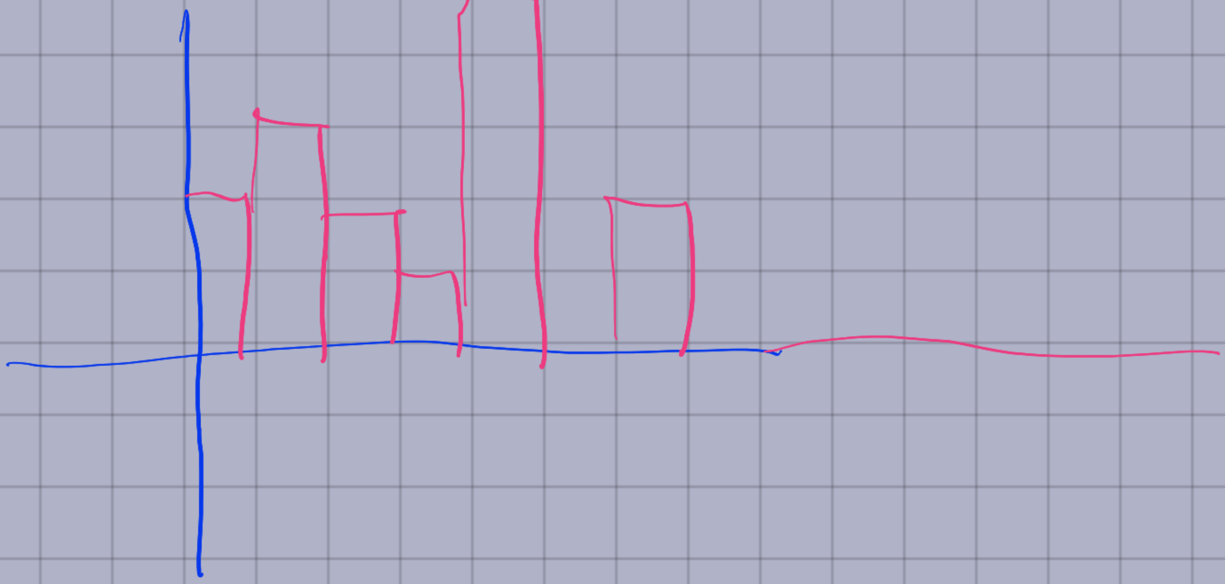
$= \underline{e}\, X$ med $X \in Mat(1,n)$

Vanligt identifiera

$$\mathbb{R}^n = Mat(1,n)$$

$$(x_1, x_2, x_3, x_4) = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix}$$

Ex $\mathbb{R}^{\infty} = (a_1, a_2, \dots, a_N, 0, 0, \dots)$



Ex $\mathbb{R}^{\mathbb{N}} = (a_0, a_1, \dots) = (a_k)_{k=0}^{\infty}$ följd.

Addition: komponentvis, skalning

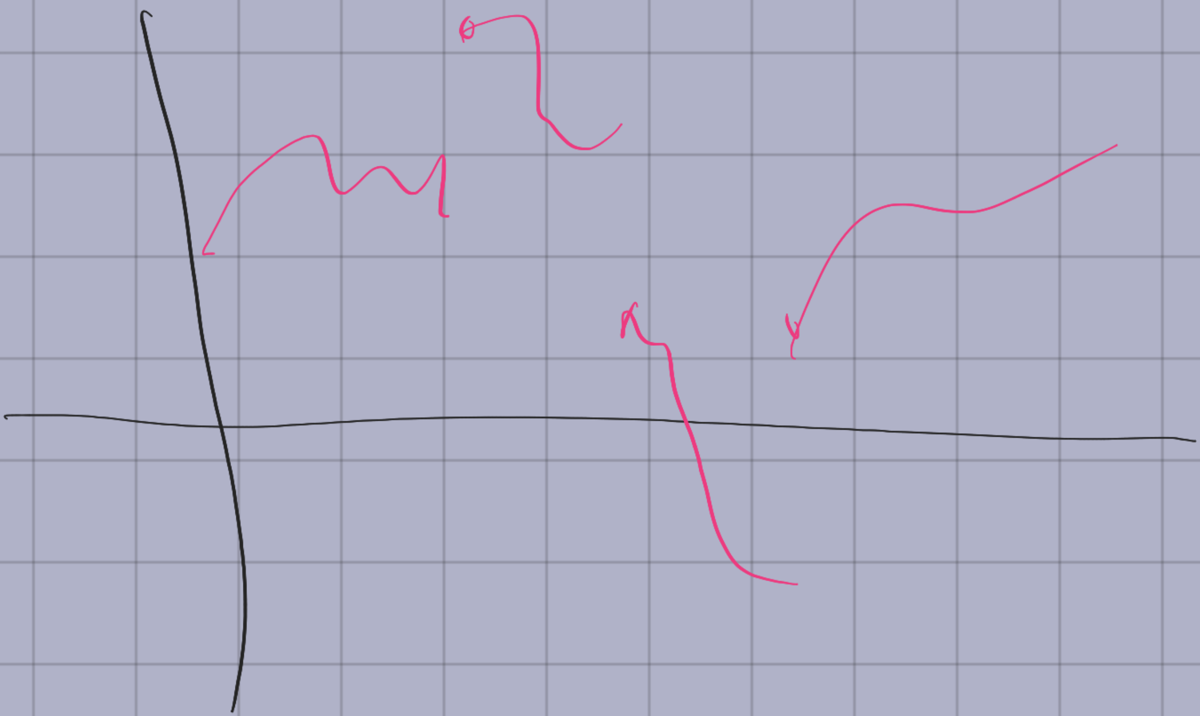
Ex $\left(\frac{1}{k+1}\right)_{k=0}^{\infty} = \left(1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \dots\right)$

✓ kan ha ∞ många nollskilda

Ex 5 V = alla funk

$f : \mathbb{R} \to \mathbb{R}$

$\bar{0}$ = funk som är konst $= 0$



Def V vr

$U \subseteq V$ delrum

(skrivs ibld $U \leq V$)

om 1) $\bar{u}_1, \bar{u}_2 \in U \Rightarrow \bar{u}_1 + \bar{u}_2 \in U$

2) $\bar{u} \in U,\ c \in \mathbb{R} \Rightarrow c\bar{u} \in U$

Följd: $0\bar{u} = \bar{0} \in U$

tmv.

Ex $\mathbb{P}$ = alla pol i x,

$$u = 3 - 17x + 5x^4$$

$$v = 11 - 2x^2 + 5x^4$$

$$u + v = 14 - 17x - 2x^2 + 10x^4$$

$$10u = 30 - 170x + 50x^4$$

$\mathbb{P}_n$ = pol av grad $\leq n$

$$\mathbb{P}_2 \ni 1 + 5x - 11x^2$$

$$\mathbb{P}_n \subseteq \mathbb{P}$$

<u>Ex</u> $\mathbb{R}^{\infty} \subseteq \mathbb{R}^{\mathbb{N}}$

1) $\bar{0} = (0, 0, \ldots, 0, \ldots) \in \mathbb{R}^{\infty}$

2) Om $\bar{u} = (u_1, \ldots, u_n, 0, 0, \ldots)$

$\bar{v} = (v_1, \ldots, v_m, 0, 0)$

så $\bar{u} + \bar{v} = (u_1 + v_1, \ldots, u_s + v_s, 0, 0, \ldots)$

med $s = \max(m, n)$

Ex lösningsmängd till
hom. linekvsys

$$\{X \in M_{n \times 1}(\mathbb{R}) : AX = 0\}$$

ty: $AX = \bar{0} \Rightarrow A(cX) = \bar{0}$

$AX_1 = AX_2 = 0$

$$\Longrightarrow A(X_1 + X_2) = AX_1 + AX_2$$

$$= \bar{0} + \bar{0} = \bar{0}$$

Ex $U = \{(x,y,z) \in \mathbb{R}^3 : x+y=0 \text{ och } 2x-z=0\}$

Delrum till $\mathbb{R}^3$.

$$\begin{array}{ccc|c} x & y & z & \\ 1 & 1 & 0 & 0 \\ 2 & 0 & 1 & 0 \end{array} \sim \begin{array}{ccc|c} 1 & 1 & 0 & 0 \\ 0 & -2 & 1 & 0 \end{array}$$

$$\sim \begin{array}{ccc|c} 1 & 0 & 1/2 & 0 \\ 0 & 1 & -1/2 & 0 \end{array}$$

$z = t$

$y = 1/2\, t$

$x = -1/2\, t$

$U = \{t(-1/2, \tfrac{1}{2}, 1) : t \in \mathbb{R}\}$

Linje genom origo

Ex $U = \{(x,y) \in \mathbb{R}^2 : x+y=1\}$

ej delrum ty $(1,0) \in U$

$(0,1) \in U$

$(1,0)+(0,1) = (1,1) \notin U$.

Men: $\{(x,y) \in \mathbb{R}^2 : x+y=0\}$

delrum

Def V vr, $\bar{u}_1, \ldots, \bar{u}_m \in V$

$c_1, \ldots, c_m \in \mathbb{R}$

Då är

$$\sum_{\ell=1}^{m} c_\ell \bar{u}_\ell \in V$$

en linj.komb av dessa elem.

$$\left\{ \sum_{\ell=1}^{m} c_\ell \bar{u}_\ell : c_1, \ldots, c_\ell \in \mathbb{R} \right\}$$

$= [\bar{u}_1, \ldots, \bar{u}_\ell]$ linj höljet,

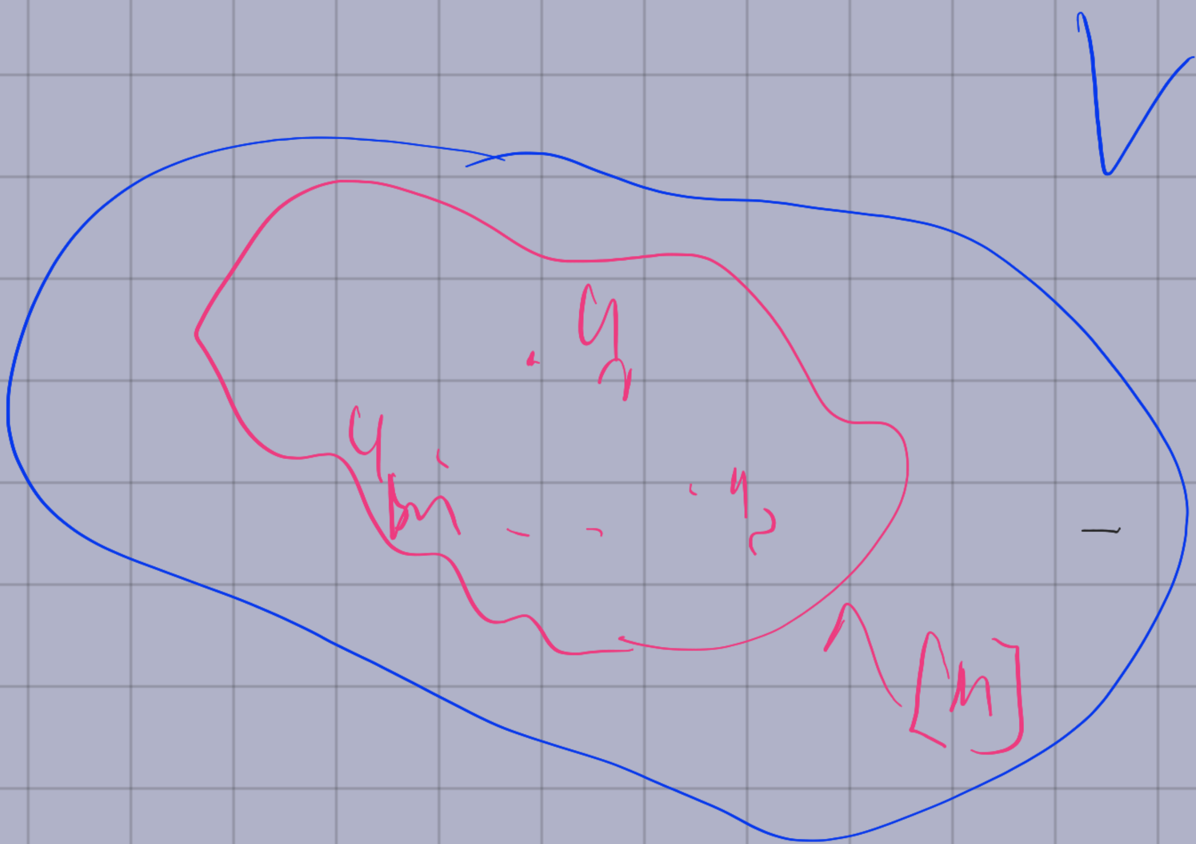
Lin $\{\bar{u}_1, \ldots, \bar{u}_m\}$

Sats. $[\bar{u}_1, \dots, \bar{u}_m] \leq V$

$\{u_1, \dots, u_m\}^{0,}$

- Om $\{\bar{u}_1, \dots, \bar{u}_m\} \subseteq W \leq V$

så $[\bar{u}_1, \dots, \bar{u}_m] \leq W$.



Ex i rummet:

$[\bar{u}]$ = linje genom origo i riktning $\bar{u}$

$[\bar{u}, \bar{v}]$ = plan genom origo spänt av $\bar{u}, \bar{v}$

$[\bar{u}, \bar{v}, \bar{w}]$ = hela rummet förmodligen

Ex $V = \mathbb{R}^4$

$\bar{e}_1 = (1,0,0,0)$ $\bar{e}_3 = (0,0,1,0)$

$\bar{e}_2 = (0,1,0,0)$ $\bar{e}_4 = (0,0,0,1)$

$[\{\bar{e}_1, \bar{e}_2, \bar{e}_3, \bar{e}_4\}] = \mathbb{R}^4$

Ex Vad är $V = [1+x,\ 1+3x+x^2] \leq \mathbb{P}^2$

$$c(1+x) + d(1+3x+x^2)$$

$$= a_0 + a_1 x + a_2 x^2 + a_3 x^3 + \dots + a_n x^n$$

1) $a_3 = a_4 = \dots = 0$

2) $c(1+x) + d(1+3x+x^2)$

$$= a_0 + a_1 x + a_2 x^2$$

q = ehr. sys

$$1 : \quad c + d = a_0$$

$$x : \quad c + 3d = a_1$$

$$x^2 : \quad d = a_2$$

$$\begin{array}{cc} c & d \\ \end{array}$$

$$\left[\begin{array}{cc|c} 1 & 1 & a_0 \\ 1 & 3 & a_1 \\ 0 & 1 & a_2 \end{array}\right] \sim \begin{array}{cc|c} 1 & 1 & a_0 \\ 0 & 2 & -a_0 + a_1 \\ 0 & 1 & a_2 \end{array}$$

$$\sim \begin{array}{cc|c} 1 & 0 & a_0 - a_2 \\ 0 & 0 & -a_0 + a_1 - 2a_2 \\ 0 & 1 & a_2 \end{array}$$

Sia $U = \{ a_0 + a_1 x + a_2 x^2 : -a_0 + a_1 - 2a_2 = 0 \}$

test $1 - 3x + x^2 \in U$

Satsen om löjliga elem:

Om $\bar{w} \in [\bar{v}_1, \ldots, \bar{v}_n] = U$

Så $V = [\bar{w}, \bar{v}_1, \ldots, \bar{v}_n] = U$

B] Klart $U \subseteq V$

Omvänt, (1) $\bar{w} = \sum_{j=1}^{n} c_j \bar{v}_j$

(2) $V \ni \bar{v} = \sum_{l=1}^{n} a_l \bar{v}_l + \bar{w}$

$$= \sum_{l=1}^{n} a_l \bar{v}_l + \sum_{j=1}^{n} c_j \bar{v}_j$$

$$= \sum_{l=1}^{n} (a_l + c_l) \bar{v}_l$$

Ex

$\bar{e}_1 = (1,0,0)$ osv

① $[\bar{e}_1, \bar{e}_2, \bar{e}_3] = \mathbb{R}^3$

② $\bar{f}_1 = \bar{e}_1, \quad \bar{f}_2 = \bar{e}_1 + \bar{e}_2, \quad \bar{f}_3 = \bar{e}_1 + \bar{e}_2 + \bar{e}_3$

③ $U = [\bar{f}_2, \bar{f}_3] \not\ni \bar{e}_2$

ty $c_1(\bar{e}_1 + \bar{e}_2) + c_2(\bar{e}_1 + \bar{e}_2 + \bar{e}_3) = \bar{e}_2$

$$\iff \left.\begin{array}{l} c_1 + c_2 = 0 \\ c_1 + c_2 = 1 \\ c_2 = 0 \end{array}\right\} \text{olösbart}$$

(4) $V = [\bar{f}_1, \hat{f}_2, \bar{e}_3] \to \bar{e}_2$

ty $\hat{e}_2 = \bar{f}_2 - \bar{f}_1$. !

(5) eftersom $U \subsetneq V$

sin $\bar{f}_1 \notin U$.