Exempel m.m. Föreläsning 14

Ex 1:

a) $A = \begin{pmatrix} | & | \\ Y_1 & Y_2 \\ | & | \end{pmatrix}$ där $\underline{v}\,Y_i = F(\bar{u}_i)$ så

$$\underline{\underline{A = \begin{pmatrix} 1 & 2 \\ 3 & 1 \end{pmatrix}}}.$$

b) Alt 1: $F\left(\underline{u}\begin{pmatrix} -1 \\ 2 \end{pmatrix}\right) = F(-\bar{u}_1 + 2\bar{u}_2) =$

$= -F(\bar{u}_1) + 2F(\bar{u}_2) = -\underline{v}\begin{pmatrix} 1 \\ 3 \end{pmatrix} + 2\underline{v}\begin{pmatrix} 2 \\ 1 \end{pmatrix} = \underline{\underline{\underline{v}\begin{pmatrix} 3 \\ -1 \end{pmatrix}}}$

Alt 2: $F(\underline{u}X) = \underline{v}AX$ ger

$F\left(\underline{u}\begin{pmatrix} -1 \\ 2 \end{pmatrix}\right) = \underline{v}\begin{pmatrix} 1 & 2 \\ 3 & 1 \end{pmatrix}\begin{pmatrix} -1 \\ 2 \end{pmatrix} = \underline{\underline{\underline{v}\begin{pmatrix} 3 \\ -1 \end{pmatrix}}}$

Ex 2:

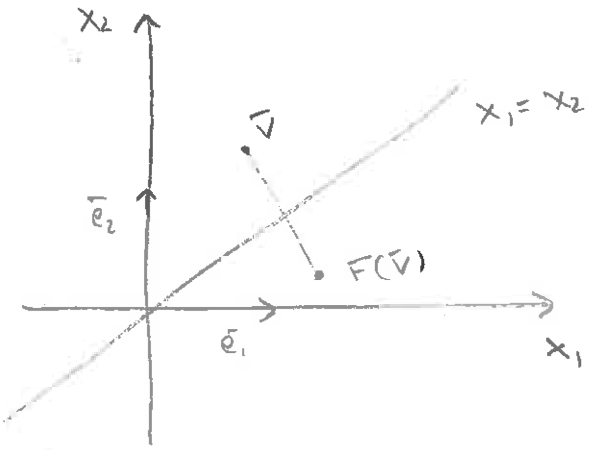
Alt 1: $F(x_1, x_2, x_3) = F\left(\underline{e}_3\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}\right) = (x_1 - 2x_2 + 4x_3,\ x_2 + 3x_3) = \underline{e}_2\begin{pmatrix} x_1 - 2x_2 + 4x_3 \\ x_2 + 3x_3 \end{pmatrix} =$

$= \underline{e}_2\begin{pmatrix} 1 & -2 & 4 \\ 0 & 1 & 3 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$ SVAR: $A = \begin{pmatrix} 1 & -2 & 4 \\ 0 & 1 & 3 \end{pmatrix}$.

Alt 2: $F(1,0,0) = (1,0) = \underline{e}_2\begin{pmatrix} 1 \\ 0 \end{pmatrix} = \underline{e}_2 Y_1$, $F(0,1,0) = \underline{e}_2\begin{pmatrix} -2 \\ 1 \end{pmatrix} = \underline{e}_2 Y_2$

$F(0,0,1) = \underline{e}_2\begin{pmatrix} 4 \\ 3 \end{pmatrix} = \underline{e}_2 Y_3$; $A = \begin{pmatrix} | & | & | \\ Y_1 & Y_2 & Y_3 \\ | & | & | \end{pmatrix} = \underline{\underline{\begin{pmatrix} 1 & -2 & 4 \\ 0 & 1 & 3 \end{pmatrix}}}.$

<u>Ex3:</u> $F: \mathbb{R}^2 \to \mathbb{R}^2$ spegling genom linjen $x_1 = x_2$. (standardbasen)



Vi noterar att

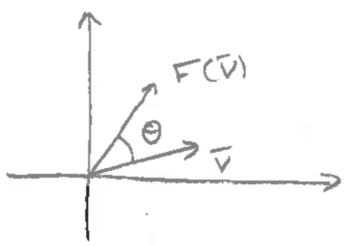
$$F(\bar{e}_1) = \bar{e}_2 = \underline{e}\begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$F(\bar{e}_2) = \bar{e}_1 = \underline{e}\begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

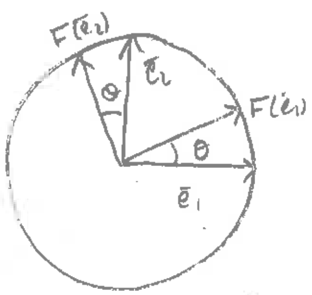
så $A = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ är F:s matris i standardbasen

Vi få alltså $F(x_1, x_2) = F\left(\underline{e}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix}\right) = \underline{e}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \underline{e}\begin{pmatrix} x_2 \\ x_1 \end{pmatrix} = (x_2, x_1)$

<u>Ex4:</u> <u>Rotation i $\mathbb{R}^2$.</u>



$F: \mathbb{R}^2 \to \mathbb{R}^2$ sådan att varje vektor $\bar{v}$ vrids med en fix vinkel θ.



$$F(\bar{e}_1) = (\cos\theta, \sin\theta)$$

$$F(\bar{e}_2) = (-\sin\theta, \cos\theta)$$

Så $A = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$

$(\det A = \cos^2\theta + \sin^2\theta = 1)$.

Ex 5: Vridning kring x_3-axeln i $\mathbb{R}^3$.

$$F(\bar{e}_1) = \underline{e}\begin{pmatrix}\cos\theta\\ \sin\theta\\ 0\end{pmatrix}, \quad F(\bar{e}_2) = \underline{e}\begin{pmatrix}-\sin\theta\\ \cos\theta\\ 0\end{pmatrix}, \quad F(\bar{e}_3) = \underline{e}\begin{pmatrix}0\\ 0\\ 1\end{pmatrix}$$

Så
$$A = \begin{pmatrix}\cos\theta & -\sin\theta & 0\\ \sin\theta & \cos\theta & 0\\ 0 & 0 & 1\end{pmatrix} \quad (\det A = 1).$$

Ex 6: Alt 1: $F(c_0 + c_1 x + c_2 x^2 + c_3 x^3) = F\left(\underline{p}\begin{pmatrix}c_0\\ c_1\\ c_2\\ c_3\end{pmatrix}\right) =$

$$= 2c_0 + 3(c_1 + c_2)x - c_3 x^2 + c_2 x^3 =$$

$$= \underline{p}\begin{pmatrix}2c_0\\ 3(c_1+c_2)\\ -c_3\\ c_2\end{pmatrix} = \underline{p}\underbrace{\begin{pmatrix}2 & 0 & 0 & 0\\ 0 & 3 & 3 & 0\\ 0 & 0 & 0 & -1\\ 0 & 0 & 1 & 0\end{pmatrix}}_{A}\begin{pmatrix}c_0\\ c_1\\ c_2\\ c_3\end{pmatrix}$$

Alt 2: $F(1) = 2 = \underline{p}\begin{pmatrix}2\\ 0\\ 0\\ 0\end{pmatrix}, \quad F(x) = 3x = \underline{p}\begin{pmatrix}0\\ 3\\ 0\\ 0\end{pmatrix}$

$F(x^2) = 3x + x^3 = \underline{p}\begin{pmatrix}0\\ 3\\ 0\\ 1\end{pmatrix} \quad F(x^3) = -x^2 = \underline{p}\begin{pmatrix}0\\ 0\\ -1\\ 0\end{pmatrix}$

$$A = \begin{pmatrix}2 & 0 & 0 & 0\\ 0 & 3 & 3 & 0\\ 0 & 0 & 0 & -1\\ 0 & 0 & 1 & 0\end{pmatrix}.$$

Ex 7: π är plan med normalvektor

$\bar{n} = (1,-2,4)$ genom origo.

Vi tar först fram en formel för $\bar{v}_{\|\bar{n}}$:

$$(x_1,x_2,x_3)_{\|\bar{n}} = \frac{(x_1,x_2,x_3)\cdot(1,-2,4)}{|(1,-2,4)|^2}(1,-2,4) =$$

$$= \frac{x_1-2x_2+4x_3}{21}(1,-2,4)$$

$F(\bar{v}) = \bar{v} - \bar{v}_{\|\bar{n}}$ så

$$F(x_1,x_2,x_3) = (x_1,x_2,x_3) - \frac{x_1-2x_2+4x_3}{21}(1,-2,4) =$$

$$= \frac{1}{21}\big((21x_1, 21x_2, 21x_3) - (x_1-2x_2+4x_3, -2x_1+4x_2-8x_3, 4x_1-8x_2+16x_3)\big) =$$

$$= \frac{1}{21}\left(20x_1+2x_2-4x_3,\ 2x_1+17x_2+8x_3,\ -4x_1+8x_2+5x_3\right)$$

$$= \frac{1}{21}\underline{e}\begin{pmatrix} 20x_1+2x_2-4x_3 \\ 2x_1+17x_2+8x_3 \\ -4x_1+8x_2+5x_3 \end{pmatrix} = \underline{e}\,\underbrace{\frac{1}{21}\begin{pmatrix} 20 & 2 & -4 \\ 2 & 17 & 8 \\ -4 & 8 & 5 \end{pmatrix}}_{\text{Matris till } F.}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$$

(Test: $F(1,-2,4) = (0,0,0)$ o.k.

$F(2,1,0) = (2,1,0)$ o.k.

$F(4,0,-1) = (4,0,-1)$ o.k.)

$G(\bar{v}) = \bar{v} - 2\bar{v}_{\parallel \bar{n}}$, så p.s.s får vi

$G(x_1, x_2, x_3) =$

$$= \frac{1}{21}\left((21x_1, 21x_2, 21x_3) - (2x_1 - 4x_2 + 8x_3, -4x_1 + 8x_2 - 16x_3, 8x_1 - 16x_2 + 32x_3)\right) =$$

$$= \frac{1}{21}\,\underline{e}\begin{pmatrix} 19x_1 + 4x_2 - 8x_3 \\ 4x_1 + 13x_2 + 16x_3 \\ -8x_1 + 16x_2 - 11x_3 \end{pmatrix} = \underline{e}\,\frac{1}{21}\begin{pmatrix} 19 & 4 & -8 \\ 4 & 13 & 16 \\ -8 & 16 & -11 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$$

(Test: $F(1,-2,4) = (-1,2,-4)$ o.k.

$F(2,1,0) = (2,1,0)$ o.k

$F(4,0,-1) = (4,0,-1)$ o.k.)