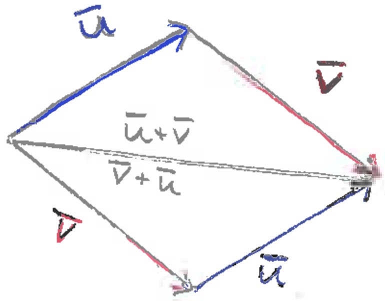
Exempel m.m. Föreläsning 2:

Förklaring av räknelagen $\bar{u}+\bar{v}=\bar{v}+\bar{u}$:



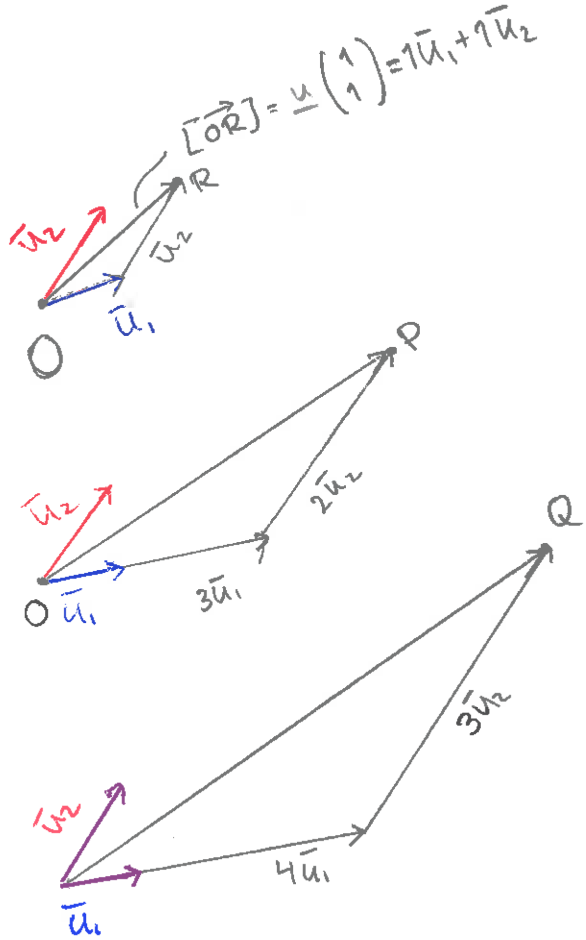
Förklaring av algebraiska operationer i koordinatsystem

$$\underline{u}\begin{pmatrix}a_1\\a_2\end{pmatrix}+\underline{u}\begin{pmatrix}b_1\\b_2\end{pmatrix}=(a_1\bar{u}_1+a_2\bar{u}_2)+(b_1\bar{u}_1+b_2\bar{u}_2)=$$

$$=(a_1+b_1)\bar{u}_1+(a_2+b_2)\bar{u}_2=\underline{u}\begin{pmatrix}a_1+b_1\\a_2+b_2\end{pmatrix}$$

$$k\,\underline{u}\begin{pmatrix}a_1\\a_2\end{pmatrix}=k(a_1\bar{u}_1+a_2\bar{u}_2)=(ka_1)\bar{u}_1+(ka_2)\bar{u}_2=\underline{u}\begin{pmatrix}ka_1\\ka_2\end{pmatrix}.$$

Lösning (Exempel 1):



$O = (0,0)$

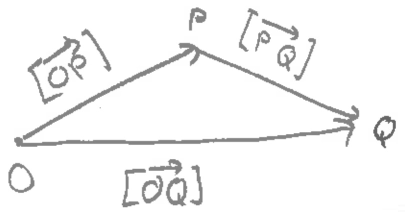
$[\overrightarrow{OR}] = \underline{u}\begin{pmatrix}1\\1\end{pmatrix}$

$[\overrightarrow{OP}] = \underline{u}\begin{pmatrix}3\\2\end{pmatrix}$

$[\overrightarrow{OQ}] = \underline{u}\begin{pmatrix}4\\3\end{pmatrix}$

(*) $[\overrightarrow{PQ}] = [\overrightarrow{OQ}] - [\overrightarrow{OP}] = \underline{u}\begin{pmatrix}4\\3\end{pmatrix} - \underline{u}\begin{pmatrix}3\\2\end{pmatrix} = \underline{u}\begin{pmatrix}1\\1\end{pmatrix}$

* Bra sätt att komma ihåg



$[\overrightarrow{OQ}] = [\overrightarrow{OP}] + [\overrightarrow{PQ}]$

$\Rightarrow \quad [\overrightarrow{PQ}] = [\overrightarrow{OQ}] - [\overrightarrow{OP}]$.

$$[\overrightarrow{OP}] + [\overrightarrow{OR}] = \underline{u}\begin{pmatrix}3\\2\end{pmatrix} + \underline{u}\begin{pmatrix}1\\1\end{pmatrix} = \underline{u}\begin{pmatrix}3+1\\2+1\end{pmatrix} = \underline{u}\begin{pmatrix}4\\3\end{pmatrix}$$

$$-[\overrightarrow{OQ}] = -1\,[\overrightarrow{OQ}] = -1\,\underline{u}\begin{pmatrix}4\\3\end{pmatrix} = \underline{u}\begin{pmatrix}-4\\-3\end{pmatrix}$$

$$4\,[\overrightarrow{OQ}] = 4\,\underline{u}\begin{pmatrix}4\\3\end{pmatrix} = \underline{u}\begin{pmatrix}4\cdot 4\\4\cdot 3\end{pmatrix} = \underline{u}\begin{pmatrix}16\\12\end{pmatrix}.$$

Lösning (Exempel 2):

$$[\overrightarrow{PR}] = [\overrightarrow{OR}] - [\overrightarrow{OP}] = \underline{u}\begin{pmatrix}-1\\0\\-2\end{pmatrix} - \underline{u}\begin{pmatrix}1\\3\\0\end{pmatrix} = \underline{u}\begin{pmatrix}-1-1\\0-3\\-2-0\end{pmatrix} = \underline{u}\begin{pmatrix}-2\\-3\\-2\end{pmatrix}.$$

$$2[\overrightarrow{PR}] + 4[\overrightarrow{OQ}] = 2\underline{u}\begin{pmatrix}-2\\-3\\-2\end{pmatrix} + 4\underline{u}\begin{pmatrix}2\\1\\3\end{pmatrix} = \underline{u}\begin{pmatrix}-4\\-6\\-4\end{pmatrix} + \underline{u}\begin{pmatrix}8\\4\\12\end{pmatrix} =$$

$$= \underline{u}\begin{pmatrix}4\\-2\\8\end{pmatrix}.$$

Lösning (Exempel 3):

(a) Vi ska avgöra om det finns k s.a.

$$\underline{u}\begin{pmatrix}3\\1\\0\end{pmatrix} = k\,\underline{u}\begin{pmatrix}-6\\-2\\0\end{pmatrix}$$

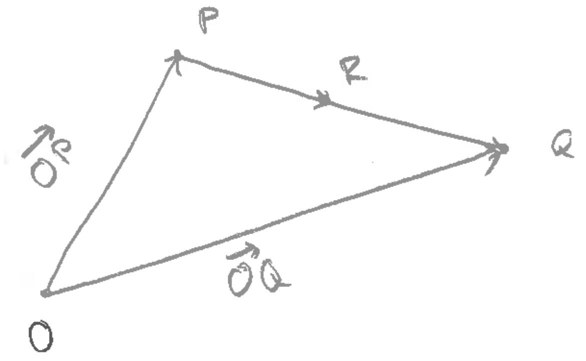
$$\Leftrightarrow \begin{cases}3 = -6k\\1 = -2k\\0 = 0k\end{cases} \Leftrightarrow \begin{cases}k = -1/2\\k = -1/2\\0 = 0\end{cases}$$

SVAR: ja.

(b) $$\underline{u}\begin{pmatrix}3\\1\\0\end{pmatrix} = k\,\underline{u}\begin{pmatrix}3\\4\\0\end{pmatrix} \Leftrightarrow \begin{cases}3 = 3k\\1 = 4k\\0 = 0\end{cases} \Leftrightarrow \begin{cases}k = 1\\k = 1/4\\0 = 0\end{cases}$$

SVAR: Nej

Lösning (Exempel 4):



$$[\vec{PR}] = \frac{1}{2}[\vec{PQ}] = \frac{1}{2}\left([\vec{OQ}] - [\vec{OP}]\right)$$

$$= \frac{1}{2}\left(\underline{u}\begin{pmatrix}2\\3\\-1\end{pmatrix} - \underline{u}\begin{pmatrix}1\\2\\4\end{pmatrix}\right) = \frac{1}{2}\,\underline{u}\begin{pmatrix}1\\1\\-5\end{pmatrix} = \underline{u}\begin{pmatrix}1/2\\1/2\\-5/2\end{pmatrix}$$

$$[\vec{OR}] = [\vec{OP}] + [\vec{PR}] = \underline{u}\begin{pmatrix}1\\2\\4\end{pmatrix} + \underline{u}\begin{pmatrix}1/2\\1/2\\-5/2\end{pmatrix} =$$

$$= \underline{\underline{\underline{u}\begin{pmatrix}3/2\\5/2\\3/2\end{pmatrix}}}.$$

Lösning (Exempel 5):

Per definition ska vi visa att för varje vektor $\bar{v} = (v_1, v_2, v_3)$ i $\mathbb{R}^3$ finns unika x_1, x_2, x_3 s.a.

$$(v_1, v_2, v_3) = \underline{u}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = x_1(1,1,3) + x_2(3,1,0) + x_3(1,0,1) =$$

$$= (x_1 + 3x_2 + x_3,\ x_1 + x_2,\ 3x_1 + x_3)$$

$$\Leftrightarrow \begin{cases} x_1 + 3x_2 + x_3 = v_1 \\ x_1 + x_2 = v_2 & (-1)\ (-3) \\ 3x_1 + x_3 = v_3 \end{cases} \Leftrightarrow \begin{cases} x_1 + x_2 = v_2 \\ 2x_2 + x_3 = v_1 - v_2 & (3/2) \\ -3x_2 + x_3 = v_3 - 3v_2 \end{cases}$$

$$\Leftrightarrow \begin{cases} x_1 + x_2 = v_2 \\ 2x_2 + x_3 = v_1 - v_2 \\ \dfrac{5x_3}{2} = \dfrac{3v_1 - 9v_2 + 2v_3}{2} \end{cases} .$$

Från denna trappstegsform ser vi att vi alltid har exakt en lösning, så $\underline{u}$ är en bas.

Med $\bar{v} = (2, 0, -3)$ får vi

$$\begin{cases} x_1 + x_2 = 0 \\ 2x_2 + x_3 = 2 \\ 5x_3 = 0 \end{cases} \Leftrightarrow \begin{cases} x_1 = -1 \\ x_2 = 1 \\ x_3 = 0 \end{cases} , \text{ så}$$

$$\underline{\underline{(2,0,-3) = \underline{u}\begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix}}}.$$