Svar, TATA24, 2021-03-14

1. $\frac{3}{\sqrt{21}}(2,4,-1)$

2. $\begin{pmatrix} 1 & 3/2 & -3 \\ -1 & -2 & 4 \\ 1 & 2 & -3 \end{pmatrix}$

3. $x_1 + 2x_2 + 6x_3 = -1$

4. $-1, 6$

5. $(-6,-1,5)$

6. 11

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7. Vi "önskar" $\begin{cases} (-1)k + m = 0 \\ 0k + m = 4 \\ 2k + m = 2 \\ 3k + m = 2 \end{cases}$, dvs $\underbrace{\begin{pmatrix} -1 & 1 \\ 0 & 1 \\ 2 & 1 \\ 3 & 1 \end{pmatrix}}_{A} \underbrace{\begin{pmatrix} k \\ m \end{pmatrix}}_{X} = \underbrace{\begin{pmatrix} 0 \\ 4 \\ 2 \\ 2 \end{pmatrix}}_{B}$.

Normalekvationerna $A^tAX = A^tB$: $\begin{pmatrix} 14 & 4 \\ 4 & 4 \end{pmatrix}\begin{pmatrix} k \\ m \end{pmatrix} = \begin{pmatrix} 10 \\ 8 \end{pmatrix}$,

vilket ger $k = \frac{1}{5}$, $m = \frac{9}{5}$. Svar: $y = \frac{1}{5}x + \frac{9}{5}$.

8. $N(F) \subseteq \mathbb{R}^4$ ges av $\left(\begin{array}{cccc|c} 2 & -3 & 0 & 2 & 0 \\ 1 & 0 & -4 & 0 & 0 \\ 2 & 6 & 1 & -4 & 0 \end{array}\right) \sim \left(\begin{array}{cccc|c} 1 & 0 & -4 & 0 & 0 \\ 0 & -3 & 8 & 2 & 0 \\ 0 & 6 & 9 & -4 & 0 \end{array}\right)$

$\sim \left(\begin{array}{cccc|c} 1 & 0 & -4 & 0 & 0 \\ 0 & -3 & 8 & 2 & 0 \\ 0 & 0 & 25 & 0 & 0 \end{array}\right)$, så $N(F) = \left[\underline{e}\begin{pmatrix} 0 \\ 2 \\ 0 \\ 3 \end{pmatrix}\right]$.

Projektionsformeln ger att $G(\bar{e}_i) = \frac{\bar{e}_i \cdot (0,2,0,3)}{13}\underline{e}\begin{pmatrix} 0 \\ 2 \\ 0 \\ 3 \end{pmatrix}$, $i = 1,2,3,4$,

så: Svar: $\frac{1}{13}\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 4 & 0 & 6 \\ 0 & 0 & 0 & 0 \\ 0 & 6 & 0 & 9 \end{pmatrix}$.

9. Sätt $A = \begin{pmatrix} 4 & -2 \\ 3 & -3 \end{pmatrix}$ så fås $\begin{pmatrix} a_n \\ b_n \end{pmatrix} = A\begin{pmatrix} a_{n-1} \\ b_{n-1} \end{pmatrix} = \ldots = A^n\begin{pmatrix} a_0 \\ b_0 \end{pmatrix}$.

Diagonalisering av A:

$\begin{vmatrix} 4-\lambda & -2 \\ 3 & -3-\lambda \end{vmatrix} = \lambda^2 - \lambda - 6 = 0$ ger $\lambda = -2, 3$.

$\lambda = 3$: $\left(\begin{array}{cc|c} 1 & -2 & 0 \\ 3 & -6 & 0 \end{array}\right)$ ger $t\begin{pmatrix} 2 \\ 1 \end{pmatrix}$, $t \in \mathbb{R}$.

$\lambda = -2$: $\left(\begin{array}{cc|c} 6 & -2 & 0 \\ 3 & -1 & 0 \end{array}\right)$ ger $t\begin{pmatrix} 1 \\ 3 \end{pmatrix}$, $t \in \mathbb{R}$.

Med $T = \begin{pmatrix} 2 & 1 \\ 1 & 3 \end{pmatrix}$ fås $A = T\begin{pmatrix} 3 & 0 \\ 0 & -2 \end{pmatrix}T^{-1}$ vilket ger

$\begin{pmatrix} a_n \\ b_n \end{pmatrix} = T\begin{pmatrix} 3^n & 0 \\ 0 & (-2)^n \end{pmatrix}\frac{1}{5}\begin{pmatrix} 3 & -1 \\ -1 & 2 \end{pmatrix}\begin{pmatrix} 1 \\ 2 \end{pmatrix} = \frac{1}{5}\begin{pmatrix} 2\cdot 3^n + 3(-2)^n \\ 3^n + 9(-2)^n \end{pmatrix}$.

Svar: $\begin{cases} a_n = \frac{2}{5}\cdot 3^n + \frac{3}{5}(-2)^n, \\ b_n = \frac{1}{5}\cdot 3^n + \frac{9}{5}(-2)^n. \end{cases}$

10. U som lösningsrum: för vilka a_0, a_1, a_2, a_3 finns c_1, c_2, c_3 så att $c_1(1+2x^3)+c_2(1-x^2+3x^3)+c_3(1+x^2+x^3) = a_0+a_1x+a_2x^2+a_3x^3$?

$$\left(\begin{array}{ccc|c} 1 & 1 & 1 & a_0 \\ 0 & 0 & 0 & a_1 \\ 0 & -1 & 1 & a_2 \\ 2 & 3 & 1 & a_3 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & 1 & 1 & a_0 \\ 0 & 0 & 0 & a_1 \\ 0 & -1 & 1 & a_2 \\ 0 & 1 & -1 & -2a_0+a_3 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & 1 & 1 & a_0 \\ 0 & 0 & 0 & a_1 \\ 0 & -1 & 1 & a_2 \\ 0 & 0 & 0 & -2a_0+a_2+a_3 \end{array}\right),$$

så $U = \left\{ a_0+a_1x+a_2x^2+a_3x^3 ;\ \begin{cases} a_1 = 0 \\ -2a_0+a_2+a_3 = 0 \end{cases} \right\}$.

Pss fås ... $V = \{ a_0+a_1x+a_2x^2+a_3x^3 ;\ 8a_0-4a_1-3a_2-a_3 = 0 \}$.

$U \cap V$ ges nu av

$$\begin{array}{c} \begin{array}{cccc} a_0 & a_1 & a_2 & a_3 \end{array} \\ \left(\begin{array}{cccc|c} -2 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 8 & -4 & -3 & -1 & 0 \end{array}\right) \end{array} \sim \left(\begin{array}{cccc|c} -2 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 3 & 0 \end{array}\right),$$

dvs $\begin{pmatrix} a_0 \\ a_1 \\ a_2 \\ a_3 \end{pmatrix} = t \begin{pmatrix} -1 \\ 0 \\ -3 \\ 1 \end{pmatrix}$.

<u>Svar:</u> $-1-3x^2+x^3$.