Svar, TATA24, 2022-03-15

1. $\begin{cases} x=-2 \\ y=0 \\ z=-1 \end{cases}$ 2. $\arccos \frac{13}{30}$ 3. $a=8$

4. $1, -3$ 5. $\begin{pmatrix} -3 & 1 \\ -2 & 1 \\ -3 & 2 \end{pmatrix}$ 6. T.ex. $T=\begin{pmatrix} 1 & 2 \\ 1 & 1 \end{pmatrix}$, $D=\begin{pmatrix} -1 & 0 \\ 0 & 2 \end{pmatrix}$

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7. U_1 som lösningsrum: $\left(\begin{array}{cccc|c} 1 & 1 & 3 & x_1 \\ -1 & 1 & 2 & x_2 \\ 2 & 2 & 0 & x_3 \\ 0 & -2 & 1 & x_4 \end{array}\right) \sim \ldots \sim \left(\begin{array}{ccc|c} 1 & 1 & 3 & x_1 \\ 0 & 2 & 5 & x_1+x_2 \\ 0 & 0 & -6 & -2x_1+x_3 \\ 0 & 0 & 0 & -x_1+x_2+x_3+x_4 \end{array}\right)$.

$U_1 \cap U_2$ är alltså lösningsrummet till systemet

$$\left(\begin{array}{cccc|c} -1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 & 0 \\ 1 & 3 & -2 & -3 & 0 \end{array}\right) \sim \ldots \sim \left(\begin{array}{cccc|c} -1 & 1 & 1 & 1 & 0 \\ 0 & 2 & 2 & 2 & 0 \\ 0 & 0 & -5 & -6 & 0 \end{array}\right)$$ vilket ger $\begin{cases} x_1=0 \\ x_2=t \\ x_3=-6t \\ x_4=5t \end{cases}$, $t\in\mathbb{R}$.

<u>Svar:</u> En bas är $((0,1,-6,5))$.

8. $X' = AX$, där $A=\begin{pmatrix} 3 & 1 & -2 \\ -1 & 1 & 2 \\ 1 & 1 & 0 \end{pmatrix}$. Diagonalisering:

$$\begin{vmatrix} 3-\lambda & 1 & -2 \\ -1 & 1-\lambda & 2 \\ 1 & 1 & -\lambda \end{vmatrix} = \begin{vmatrix} 3-\lambda & 1 & -2 \\ 2-\lambda & 2-\lambda & 0 \\ 1 & 1 & -\lambda \end{vmatrix} = (2-\lambda)\begin{vmatrix} 2-\lambda & 0 & -2 \\ 1 & 1 & 0 \\ 0 & 0 & -\lambda \end{vmatrix} = -\lambda(2-\lambda)^2.$$

$\lambda=0$: $\left(\begin{array}{ccc|c} 3 & 1 & -2 & 0 \\ -1 & 1 & 2 & 0 \\ 1 & 1 & 0 & 0 \end{array}\right) \sim \left(\begin{array}{ccc|c} 0 & 0 & 0 & 0 \\ 0 & 2 & 2 & 0 \\ 1 & 1 & 0 & 0 \end{array}\right)$ ger $r\begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}$, $r\in\mathbb{R}$.

$\lambda=2$: $\left(\begin{array}{ccc|c} 1 & 1 & -2 & 0 \\ -1 & -1 & 2 & 0 \\ 1 & 1 & -2 & 0 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & 1 & -2 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right)$ ger $r\begin{pmatrix} 0 \\ 2 \\ 1 \end{pmatrix} + s\begin{pmatrix} 2 \\ 0 \\ 1 \end{pmatrix}$, $r,s\in\mathbb{R}$.

<u>Svar:</u> $\begin{pmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{pmatrix} = C_1\begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix} + C_2 e^{2t}\begin{pmatrix} 0 \\ 2 \\ 1 \end{pmatrix} + C_3 e^{2t}\begin{pmatrix} 2 \\ 0 \\ 1 \end{pmatrix}$.

9. Gram-Schmidt utgående från $(1,0)$, $(0,1)$:

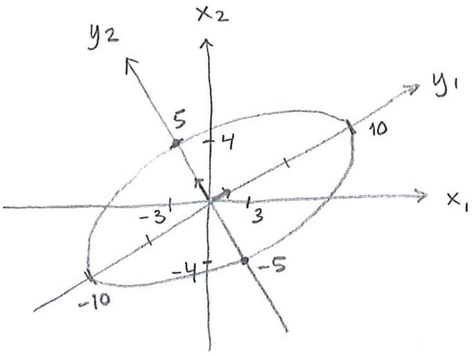
$\bar{g}_1 = (1,0)$. $\quad (\bar{g}_1|\bar{g}_1) = 2\cdot 1^2 - 4(1\cdot 0 + 0\cdot 1) + 9\cdot 0^2 = 2$.

$$\bar{g}_2 = (0,1) - \frac{((0,1)|\bar{g}_1)}{(\bar{g}_1|\bar{g}_1)}\bar{g}_1 = (0,1) - \frac{2\cdot 0 - 4(0+1) + 9\cdot 0}{2}(1,0) = (2,1)$$

$(\bar{g}_2|\bar{g}_2) = 2\cdot 2^2 - 4(2\cdot 1 + 1\cdot 2) + 9\cdot 1^2 = 1$. Normering ger:

Svar: $(\bar{f}_1\ \bar{f}_2) = \left(\frac{1}{\sqrt{2}}(1,0)\ \ (2,1)\right)$.

10.



$\sqrt{(-3)^2+4^2} = 5$. $\quad (4,3) \perp (-3,4)$.

Inför en ny ON-bas

$$(\bar{f}_1\ \bar{f}_2) = ((1,0)\ (0,1))\underbrace{\begin{pmatrix} 4/5 & -3/5 \\ 3/5 & 4/5 \end{pmatrix}}_{T}$$

Koordinatsamband: $Y = T^{-1}X = T^tX$.

I nya koord. y_1, y_2 har ellipsen ekvationen $\frac{y_1^2}{10^2} + \frac{y_2^2}{5^2} = 1$.

Detta ger:
$$\frac{\left(\frac{1}{5}(4x_1+3x_2)\right)^2}{10^2} + \frac{\left(\frac{1}{5}(-3x_1+4x_2)\right)^2}{5^2} = 1.$$

Efter förenkling fås:

Svar: $52x_1^2 - 72x_1x_2 + 73x_2^2 = 2500$.