Beräkna de obestämda integralerna

(a) $\displaystyle\int \frac{dx}{x^2-4x+3}$ (b) $\displaystyle\int \frac{x+3}{x^2+2x+5}\,dx$ (c) $\displaystyle\int \frac{\sin^3(\sqrt{x})}{\sqrt{x}}\,dx$

a) $\displaystyle\int \frac{dx}{x^2-4x+3} =$

$= \Big/ \; x^2-4x+3 = (x-2)^2-4+3 = (x-2)^2-1^2 = (x-2+1)(x-2-1) = (x-1)(x-3)$

PBU: $\displaystyle\frac{1}{(x-1)(x-3)} = \frac{A}{x-1} + \frac{B}{x-3}$ Handpåläggning: $A = -\frac{1}{2}$, $B = \frac{1}{2}$ $\Big/$

$$= \int \left(\frac{-1/2}{x-1} + \frac{1/2}{x-3} \right) dx = \frac{1}{2}\ln|x-3| - \frac{1}{2}\ln|x-1| + C$$

b) $\displaystyle\int \frac{x+3}{x^2+2x+5}\,dx = \Big/ \; x^2+2x+5 = (x+1)^2+4, \quad t = x+1, \quad \frac{dt}{dx} = 1 \text{ så } dt = dx \Big/ = \int \frac{t-1+3}{t^2+4}\,dt$

$$= \int \left(\frac{t}{t^2+4} + 2\frac{1}{t^2+4} \right) dt = \int \left(\frac{t}{t^2+4} + \frac{1}{2}\frac{1}{(\frac{t}{2})^2+1} \right) dt$$

$$= \frac{1}{2}\ln|t^2+4| + \frac{1}{2}\cdot 2\arctan\frac{t}{2} + C$$

$$= \frac{1}{2}\ln(x^2+2x+5) + \arctan\frac{x+1}{2} + C$$

c) $\displaystyle\int \frac{\sin^3(\sqrt{x})}{\sqrt{x}}\,dx = \Big/ \; t = \sqrt{x}, \quad \frac{dt}{dx} = \frac{1}{2\sqrt{x}} \text{ dvs } 2dt = \frac{1}{\sqrt{x}}dx \Big/ = \int 2\sin^3 t\,dt =$

$$= \int 2(1-\cos^2 t)\sin t\,dt = \Big/ \; s = \cos t, \quad \frac{ds}{dt} = -\sin t \text{ dvs } -ds = \sin t\,dt \Big/$$

$$= -2\int (1-s^2)\,ds = -2\left(s - \frac{1}{3}s^3\right) + C = \frac{2}{3}\cos^3(\sqrt{x}) - 2\cos\sqrt{x} + C$$

Svar: a) $\frac{1}{2}\ln|x-3| - \frac{1}{2}\ln|x-1| + C$

b) $\frac{1}{2}\ln(x^2+2x+5) + \arctan\frac{x+1}{2} + C$

c) $\frac{2}{3}\cos^3(\sqrt{x}) - 2\cos\sqrt{x} + C$