Exempel: Beräkna $\sum_{n=1}^{\infty} n^2 3^{-n}$.

Lösning:

$$f(x) = \sum_{n=0}^{\infty} x^n = \frac{1}{1-x} \qquad |x|<1.$$

$$f'(x) = \sum_{n=1}^{\infty} n x^{n-1} = \frac{1}{(1-x)^2} \qquad |x|<1$$

$$f''(x) = \sum_{n=2}^{\infty} \underbrace{n(n-1)x^{n-2}}_{n^2 x^{n-2} \underbrace{- n x^{n-2}}} = \frac{1}{(1-x)^3} \qquad |x|<1.$$

$$\sum_{n=2}^{\infty} n^2 x^n = x^2 \sum_{n=2}^{\infty} n^2 x^{n-2} = x^2\left(f''(x) + \sum_{n=2}^{\infty} n x^{n-2} \right) =$$

$$= /\text{om } |x|<1,\ x \neq 0/ = x^2\left(f''(x) + \frac{1}{x} \underline{\sum_{n=2}^{\infty} n x^{n-1}} \right)$$

$$= x^2\left(f''(x) + \frac{1}{x}\left(\underline{f'(x) - 1} \right) \right) =$$

$$= x^2 \left(\frac{2}{(1-x)^3} + \frac{1}{x}\left(\frac{1}{(1-x)^2} - 1 \right) \right) \qquad 0 < |x| < 1 .$$

Med $x = 1/3$ får vi

$$\sum_{n=2}^{\infty} n^2 3^{-n} = \sum_{n=2}^{\infty} n^2 \left(\frac{1}{3}\right)^n =$$

$$= \left(\frac{1}{3}\right)^2 \left(\frac{2}{(1-1/3)^3} + \frac{1}{1/3}\left(\frac{1}{(1-1/3)^2} - 1\right)\right) = \ldots = \underline{\underline{\frac{7}{6}}}.$$