Exempel

Beräkna

$$\lim_{x \to 0} \frac{6(\arctan(x) - x)(\cos(x) - 1) - x^5}{\sin(x^7)}.$$

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ger med $t = x^7$

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Steg 2: Utveckla täljaren så att vi får $\mathcal{O}(x^8)$ eller bättre:

$$6(\arctan(x) - x)(\cos(x) - 1) - x^5 =$$

$$6(x - \frac{x^3}{3} + \frac{x^5}{5} + \mathcal{O}(x^7) - x)(1 - \frac{x^2}{2} + \frac{x^4}{4!} + \mathcal{O}(x^6) - 1) - x^5.$$

$$\frac{6(\arctan(x) - x)(\cos(x) - 1) - x^5}{\sin(x^7)} =$$

$$\frac{6(\arctan(x)-x)(\cos(x)-1)-x^5}{\sin(x^7)} =$$

$$\frac{6(x-\frac{x^3}{3}+\frac{x^5}{5}+\mathcal{O}(x^7)-x)(1-\frac{x^2}{2}+\frac{x^4}{4!}+\mathcal{O}(x^6)-1)-x^5}{x^7+\mathcal{O}(x^{21})} =$$

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$$\frac{6(-\frac{x^3}{3}+\frac{x^5}{5}+\mathcal{O}(x^7))(-\frac{x^2}{2}+\frac{x^4}{4!}+\mathcal{O}(x^6))-x^5}{x^7+\mathcal{O}(x^{21})} =$$

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$$\frac{6(-\frac{x^3}{3}+\frac{x^5}{5}+\mathcal{O}(x^7))(-\frac{x^2}{2}+\frac{x^4}{4!}+\mathcal{O}(x^6))-x^5}{x^7+\mathcal{O}(x^{21})}=$$

$$\frac{6\left(\frac{-x^3}{3}\cdot\frac{-x^2}{2}+\frac{-x^3}{3}\cdot\frac{x^4}{4!}+\frac{x^5}{5}\cdot\frac{-x^2}{2}+\mathcal{O}(x^9)\right)-x^5}{x^7+\mathcal{O}(x^{21})}=$$

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$$\frac{6(-\frac{x^3}{3}+\frac{x^5}{5}+\mathcal{O}(x^7))(-\frac{x^2}{2}+\frac{x^4}{4!}+\mathcal{O}(x^6))-x^5}{x^7+\mathcal{O}(x^{21})} =$$

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$$\frac{\left(x^5-\frac{41x^7}{60}+\mathcal{O}(x^9)\right)-x^5}{x^7+\mathcal{O}(x^{21})} =$$

$$\frac{6(\arctan(x) - x)(\cos(x) - 1) - x^5}{\sin(x^7)} =$$

$$\frac{6(x - \frac{x^3}{3} + \frac{x^5}{5} + \mathcal{O}(x^7) - x)(1 - \frac{x^2}{2} + \frac{x^4}{4!} + \mathcal{O}(x^6) - 1) - x^5}{x^7 + \mathcal{O}(x^{21})} =$$

$$\frac{6(-\frac{x^3}{3} + \frac{x^5}{5} + \mathcal{O}(x^7))(-\frac{x^2}{2} + \frac{x^4}{4!} + \mathcal{O}(x^6)) - x^5}{x^7 + \mathcal{O}(x^{21})} =$$

$$\frac{6\left(\frac{-x^3}{3} \cdot \frac{-x^2}{2} + \frac{-x^3}{3} \cdot \frac{x^4}{4!} + \frac{x^5}{5} \cdot \frac{-x^2}{2} + \mathcal{O}(x^9)\right) - x^5}{x^7 + \mathcal{O}(x^{21})} =$$

$$\frac{\left(x^5 - \frac{41x^7}{60} + \mathcal{O}(x^9)\right) - x^5}{x^7 + \mathcal{O}(x^{21})} =$$

$$\frac{-\frac{41}{60} + \mathcal{O}(x^2)}{1 + \mathcal{O}(x^{14})} \to -\frac{41}{60} \text{ då } x \to 0.$$