Exempel

Beräkna längden av parameterkurvan:

$$\begin{cases} x = t^2 + 1 \\ y = t^3 - 1, \quad 1 \leq t \leq 2. \end{cases}$$

Lösning

Lösning

$$ds = \sqrt{x'(t)^2 + y'(t)^2}dt =$$

Lösning

$$ds = \sqrt{x'(t)^2 + y'(t)^2}dt =$$

$$\sqrt{(2t)^2 + (3t^2)^2}dt = \sqrt{4t^2 + 9t^4}dt.$$

Lösning

$$ds = \sqrt{x'(t)^2 + y'(t)^2}dt =$$

$$\sqrt{(2t)^2 + (3t^2)^2}dt = \sqrt{4t^2 + 9t^4}dt.$$

$$\int_1^2 ds = \int_1^2 \sqrt{4t^2 + 9t^4}dt =$$

Lösning

$$ds = \sqrt{x'(t)^2 + y'(t)^2}dt =$$

$$\sqrt{(2t)^2 + (3t^2)^2}dt = \sqrt{4t^2 + 9t^4}dt.$$

$$\int_1^2 ds = \int_1^2 \sqrt{4t^2 + 9t^4}dt =$$

$$\int_1^2 t\sqrt{4 + 9t^2}dt =$$

Lösning

$$ds = \sqrt{x'(t)^2 + y'(t)^2} dt =$$

$$\sqrt{(2t)^2 + (3t^2)^2} dt = \sqrt{4t^2 + 9t^4} dt.$$

$$\int_1^2 ds = \int_1^2 \sqrt{4t^2 + 9t^4} dt =$$

$$\int_1^2 t\sqrt{4 + 9t^2} dt =$$

$$\left[\frac{1}{27}(4 + 9t^2)^{3/2}\right]_1^2 =$$

$$\frac{40^{3/2} - 13^{3/2}}{27}.$$

Svar: $\frac{40^{3/2} - 13^{3/2}}{27}$.