

Hand-in Exercises 1, TATA49 Geometry with Applications. Fall 2026

Maple, Python or Matlab may/should be used in the calculations.

The exercises are customized by your birthdays as follows: p_1, p_2 is the month of your birthday, where January is 01, February 02, ..., and December 12. q_1, q_2 is the date in the month, between 13 and 29, nearest to your birthday. r_1, r_2 are the two last digits in the year you were born. However, if you were born in 1990, then $r_1, r_2 = 9, 1$; if you are born in 200i $r_1 = 5 + i, r_2 = 4 + i$.

Exercise 1 1. Show that the lines $l[1, 1, 1]$, $m[r_1, r_2, p_1 + q_1]$, $n[q_2 + 4r_1, q_2 + 4r_2, q_2 + 4p_1 + 4q_1]$ are concurrent and determine the intersection point Int .

2. Show that the points $P(-1/2, -1/2, 1)$, $Q(-5, 10, 1)$, $R(-2, 3, 1)$ are collinear and give the equation of the line $t[t_1, t_2, t_3]$ they belong to.

3. Determine the angles at Int formed by l, m and n (of course pairwise).

4. Show that there is a triangle whose sides lie on the lines l, m, t .

Exercise 2 Determine the affine transformation, if existing, that takes the line $l[1, p_1, p_2 + 1]$ to the line $l'[2, 0, 1]$, the line $m[0, q_1, q_2 + r_1]$ to $m'[1, -2, 2]$ and $n[r_1, r_2, q_1 + q_2]$ to $n'[1, 2, -2]$.

Exercise 3 Determine the affine transformation that takes the point $P(1, p_1, 1)$ to $P'(2, 0, 1)$, the point $Q(q_1, q_2 + r_1, 1)$ to $Q'(1/2, -1, 1)$ and $R(r_1 + q_1, r_2 + q_2, 1)$ to $R'[-1/2, -1, 1]$.

Exercise 4 a Determine the image of the circle with equation $(r_1^2 + r_2^2)x_1^2 + (r_1^2 + r_2^2)x_2^2 - 4x_1 - 6x_2 = 9$ under the affine transformation given in Exercise 2. Determine the eccentricities.

b Show that an affine transformation transforms an ellipse, hyperbola respectively in another ellipse (hyperbola). Recall that if a hyperbola is defined by a symmetric matrix C with eigenvalues $\lambda_1 > 0$ and $\lambda_2 < 0$, then the eccentricity e is defined by $e = \sqrt{1 + \frac{\lambda_1}{|\lambda_2|}}$. The eccentricity of an ellipse is given by $e = \sqrt{1 - \frac{\lambda_1}{|\lambda_2|}}$, where $0 < \lambda_1 < \lambda_2$. Determine the relation between eccentricities.

Exercise 5 1. Consider the square with sides on the lines $l(2, -1, 0]$, $m(1, 2, -0]$, $n[2, -1, 5]$, $r[1, 2, -5]$. Verify that these lines form a square.

2. Determine the side, vertices and inner angles of the image of the square under the affine transformation with matrix $\begin{pmatrix} r_2 & r_2 + p_2 & 2q_1 \\ r_1 + p_1 & r_1 + p_2 & 3q_2 \\ 0 & 0 & 1 \end{pmatrix}$

Exercise 6 Determine the isometry that takes the points $P(r_2 + p_1, r_1 + p_2)$ and $Q(p_1 + q_1, p_2 + q_2)$ to $P'(r_1, r_2)$ and $Q'(r_1 + \frac{q_1\sqrt{(r_2-q_1)^2+(r_1-q_2)^2}}{\sqrt{q_1^2+q_2^2}}, r_2 + \frac{q_2\sqrt{(r_2-q_1)^2+(r_1-q_2)^2}}{\sqrt{q_1^2+q_2^2}})$ respectively. **The** isometry?

Exercise 7 Let $P(2, 2, 1)$ and $Q(5, 0, 1)$ be points. Consider the triangle $P, Q, R(p_1 + r_1, p_2 + r_2)$. Determine the image of the triangle under an indirect similarity of ratio 2 and axis $s[r_1, r_2, p_2 + q_2 + r_1]$.

Exercise 8 Give the (different) indirect isometries that take the point $P(p_1 + q_1, p_2 + q_2)$ to the point $P'(r_1 + q_2, r_2 + q_1)$.

Exercise 9 Give the (different) direct isometries that takes the point $P(p_1 + q_1, p_2 + q_2)$ to the point $P'(r_1 + q_2, r_2 + q_1)$. What is the relation between the locus (set) of glide reflection axes above and the locus of rotation centres her?

Exercise 10 Determine the isometries that fix all the lines (as lines) in the family $\mathcal{L} = \{l_t[2r_1 : 3r_2 : p_1 + q_2 + 4t]; t \in \mathbb{R}\}$. Which degree of freedom does one get?

Exercise 11 A) Show that any direct isometry T of the plane can be expressed in terms of complex numbers as $T(z) = az + v$, where $|a| = 1$, $v = v_1 + iv_2$, $z = x + iy$.

B) Give the matrix of the isometry above.

Exercise 12 Give conditions to the different centres and angles of two rotations such that the product of the two rotations is a translation. What is the vector of the resulting translation?

Exercise 13 Consider the affine transformation T with matrix

$$\begin{pmatrix} 1 + abk & a^2k & 0 \\ -b^2k & 1 - abk & 0 \\ 0 & 0 & 1 \end{pmatrix},$$
 where $k > 0$, and a, b satisfy $a^2 + b^2 = 1$. Describe it geometrically. Is there an axis for the transformation? Which one?

Exercise 14 1. In the figure below you see the arm of a planar robot. Give the function f that determines the configuration (position and orientation, total angle) of the hand depending on the parameters of the arm. The length of the fixed rigid link is $l_1 = 8$ units, the length of the rigid link starting at J_2 is $l_2 = 4$, and the length of the rigid link to the hand is $l_3 = 4$.

2. With the robot arm as in the figure below, determine, if possible, the parameters for the angles at the junctions (links) the controller should get if the coordinates of the point are $P_{(H)}(-4, 2\sqrt{3})$, relative to the local coordinates (x_H, y_H) at the hand, and $P_0(-\sqrt{3}, 7)$, relative to the coordinates (x_0, y_0) for the controller. The total angle for the hand (the part of the arm with the hand) is $2\pi/3$ rad, and the segments all have length 1.

