# **Course Program**

• TATA53 • Linear Algebra Honours Course • 6 credits • Spring 2025 •

Examiner: Jonathan Nilsson jonathan.nilsson@liu.se

Website: https://courses.mai.liu.se/GU/TATA53/

#### Literature:

The material presented in the lectures will mainly follow the **course compendium** available on the course website.

As a complement I suggest Axler: *Linear algebra done right* (4'th ed. 2025), available free from the authors website: https://linear.axler.net/

Other recommended books can be found on via the course website.

### **Course contents**

Vector spaces, linear maps, inner product spaces, spectral theory, matrix factorizations, Jordan normal form with applications, singular values with applications, Perron-Frobenius theory with applications, introduction to multilinear algebra.

### Teaching

There are 15 lectures (about 1 per week) and 7 exercise sessions (about 1 per two weeks). The lectures provide an overview of the theory, and is a complement to the course literature.

## Examination

The examination consists of four sets of hand-in assignments. For each of the four assignments there is a corresponding seminar where you will present your solutions. For grade 3/4/5 you will need a total score corresponding to 60%/75%/85% on the hand-in assignments, with at least 60% on each one. You also need to be ready to present your solutions at the seminars. For grade 5 you are also expected to present solutions to a few of the more challenging problems. To allow as many students as possible to make presentations there are two instances of each seminar scheduled - you should attend one of each (so four in total).

Set	Deadline	Main topics
1	13/2 8.00am	Vector spaces, direct sums, LU-factorization
2	$10/3 \ 10.00 \mathrm{am}$	Jordan normal form, inner product spaces
3	24/4 8.00am	QR-factorization, spectral theory, Perron-Frobenius
4	$12/5 \ 10.00 \mathrm{am}$	Singular values, multilinear algebra

Each assignments will be published on the course website two weeks before the corresponding deadline. Hand in your assignments in the compartment labelled "TATA53-submissions" in house B, entrance 21, one floor up just outside the A-corridor. Alternatively you can hand them in at the beginning of the first of the two seminars (meaning 15 minutes after the deadline listed above).

Below is the schedule for Lectures and Seminars of the course. There are also 7 lessons scheduled, always refer to TimeEdit for the current version and exact schedule. For each lecture there is a list of recommended exercises from the course compendium on the course website. These are intended to prepare you for the assignment problems. It is not necessary to do all of them, but try and solve few of these after each lecture to keep up with the course.

Date		Topics/suggested exercises
20/1	Le1	Fields, vector spaces, basis, subspaces, direct sum, quotients, linear maps
		<b>1:</b> 1-10, 13, (16, 22), 24, 25; <b>9:</b> (1, 4), 6, (7), 13
27/1	Le2	Echelon forms, elementary matrices, rank, trace, LU-factorization
		<b>2:</b> 1, 2, (3), 7, 9, 12, 16, (17), 19, 21, (22, 26), 29, 31
31/1	Le3	Cholesky-factorization, eigenvectors, spectra, multiplicities, diagonalization
		<b>2:</b> 35, (36, 37); <b>3:</b> 1, 2, 3, 4, (5), 6a(b), 7, (9), 10, (11)
6/2	$\mathbf{Le}4$	Cayley-Hamilton, minimal polynomial, Jordan Normal form, nilpotent maps
		<b>3:</b> 13, (15), 16, 17, 19, 20, 21, (23); <b>4:</b> 1, 2, (3, 4, 5, 6), 10, 11, 12
13/2	Se1a	Seminar 1. Presentations of exercises. Attend 1a or 1b.
14/2	Se1b	Seminar 1. Presentations of exercises. Attend 1a or 1b.
17/2	Le5	Generalized eigenspaces, Jordan chains, Jordan theorem, Jordanization
		<b>4:</b> 13, 14, 15, 16, (17), 18, (19, 20), 21, (27)
24/2	$\mathbf{Le}6$	Matrix exponential, dynamical systems, inner product spaces
		<b>4:</b> 30, 31, 32, (35, 36), 37, 38; <b>5:</b> 1, 2, 3, 4, (6), 7
3/3	$\mathbf{Le7}$	Norms, Gram-Schmidt, QR-factorization
		<b>5:</b> 8, 11, 12, (16, 17, 18), 20, 23, (24), 25, 26, (27, 28), 29, 30
6/3	Le8	Self-adjoint, unitary, normal, and positive-definite operators, spectral theorems
		<b>5:</b> 33, 35, 36, (37, 38, 39), 40, 41, 42, (43, 44), 45, (46, 47), 48, (49), 50
10/3	Se2a	Seminar 2. Presentations of exercises. Attend 2a or 2b.
11/3	$\mathbf{Se}2\mathbf{b}$	Seminar 2. Presentations of exercises. Attend 2a or 2b.
1/4	Le9	Least squares, positive matrices, Perron's theorem
		<b>5:</b> 51, 52, (53); <b>6:</b> 1, 2, (3, 4), 5, (6, 7), 9, 10, 11, 12, (13), 14
8/4	Le10	Non-negative matrices, Frobenius' theorem, ranking models, Markov chains
		<b>6:</b> 15, 16, 17, (18, 19), 20, (22), 23, 24, 25, 26, (27, 28)
14/4	Le11	Singular values and vectors, singular value decomposition
		<b>7:</b> 1, 2, 3, (4), 5, 7, (8), 9, (10, 11), 12, 14, 15, (16, 18, 19)
17/4	Le12	Schmidt-decomposition, low rank approximation, Eckart-Young, pseudo-inverse
		<b>7:</b> 20, 21, (22, 23), 24, (25), 26, 27, (28, 29), 30, 31, (32, 34, 35)
24/4	Se3a	Seminar 3. Presentations of exercises. Attend 3a or 3b.
25/4	Se3b	Seminar 3. Presentations of exercises. Attend 3a or 3b.
28/4	<b>Le</b> 13	Condition number, polar factorization, principal component analysis
	<b>T</b> - 1	<b>7:</b> 36, 37, 38, (39), 40, 41, 42, (43), 44, 45
5/5	<b>Le</b> 14	Total least squares, multilinear algebra, tensor products, duals
	T 4 F	<b>7:</b> 46, 47, (48); <b>8:</b> 1, 2, (3), 4, 5, 8, 10, (11), 12, 13, (14), 15
8/5	<b>Le</b> 15	Higher order tensors, Kronecker product, neural networks
10/5	<b>G</b> 4	<b>8:</b> 16, (17, 20), 21, 22, 23, 25, 26, 27, 30, (31, 32), 34, (37), 40
12/5	Se4a	Seminar 4. Presentations of exercises. Attend 4a or 4b.
13/5	Se4b	Seminar 4. Presentations of exercises. Attend 4a or 4b.