

Kurskod: TATA 54

Provkod: TEN 1

NUMBER THEORY, Talteori 6 hp

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Examiner: Leif Melkersson

Inga hjälpmedel är tillåtna! (For example books or pocket calculators are not allowed!)

You may write in Swedish, if you do this consistently.

You are rewarded at most 3 points for each of the 6 problems.

To get grade 3, 4 or 5, you need respectively 7, 11 and 14 points.

- (1) Can n be written as $n = x^2 + y^2$, where x and y are integers, when
 - (a) $n = 1098$
 - (b) $n = 4067$
- (2)
 - (a) Show that $\sqrt{65} = [8; \overline{16}]$.
 - (b) Find the smallest solution (x, y) in positive integers of the diophantine equation $x^2 - 65y^2 = 1$.
- (3) Factorise the gaussian integer $45 + 60i$ into gaussian primes.
- (4) Solve the congruence $x^3 + 2x^2 + x + 1 \equiv 0 \pmod{5^2}$
- (5)
 - (a) Find a primitive root of 11.
 - (b) Make a table of indices modulo 11 with respect to this primitive root.
 - (c) Find all integers $x \geq 0$, such that $7^x \equiv 3 \pmod{11}$.
- (6) Decide if the congruence $3x^2 + x + 6 \equiv 0 \pmod{59}$ has any solutions or not.