Number theory, Talteori 6hp, Kurskod TATA54, Provkod TEN1
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Each problem is worth 3 points. To receive full points, a solution needs to be complete. Indicate which theorems from the textbook that you have used, and include all auxillary calculations.

No aids, no calculators, tables, nor texbooks.

1) Use the Chinese Remainder Theorem to find all solutions to

$$x^2 \equiv 15 \mod 77.$$

2) For which positive n does the congruence

$$x^5 + x + 1 \equiv 0 \mod 5^n$$

have a unique solution? Find all solutions for $n = 1, 2$.

3) Let $x = [13; \overline{1, 7}]$. Compute the value of x.

4) The function f satisfies

$$\begin{aligned} f(1) &= 1 \\ f(1) + f(2) &= a \\ f(1) + f(3) &= b \\ f(1) + f(2) + f(4) &= c \\ f(1) + f(2) + f(3) + f(6) &= ab \\ f(1) + f(2) + f(3) + f(4) + f(6) + f(12) &= bc \end{aligned}$$

Calculate $f(12)$. For which a, b, c can f be extended to a multiplicative function on the positive integers?

5) Show that 10 is a primitive root modulo 17. List all quadratic residues mod 17.

6) The number 41 is a prime. Show that -1 is a quadratic residue module 41, then find a solution to the congruence

$$x^2 \equiv -1 \mod 41$$

Among the solutions (m, n) to

$$mx + n \equiv 0 \mod 41$$

find a pair with $0 < |m|, |n| \le 6$. Show that $41 = m^2 + n^2$.