Number theory, Talteori 6hp, Kurskod TATA54, Provkod TEN1
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LINKÖPINGS UNIVERSITET
Matematiska Institutionen
Examinator: Jan Snellman

Each problem is worth 3 points. To receive full points, a solution needs to be complete. Indicate which theorems from the textbook that you have used, and include all auxillary calculations.

No aids, no calculators, tables, nor texbooks.

1) Find all integers x such that

$$\begin{aligned} x &\equiv 5 \mod 11 \\ 2x &\equiv 1 \mod 13. \end{aligned}$$

2) How many incongruent solutions are there to the congruence

$$5x^3 + x^2 + x + 1 \equiv 0 \mod 32?$$

3) Use the fact that 3 is a primitive root modulo 17 to find all solutions to the congruence

$$10^x \equiv 5 \mod 17.$$

4) Let $x = [1; \overline{1, 2}]$. Compute the value of x.

5) Let $p > 3$ be a prime. Show that

$$\left(\frac{3}{p}\right) = 1 \iff p \equiv \pm 1 \mod 12$$

6) Let $\omega(n)$ be the number of distinct primes that divide n, $\tau(d)$ be the number of positive divisors of d, and let μ be the Möbius function.

(a) Show that $n \mapsto 3^{\omega(n)}$ is multiplicative.

(b) Using the above, and the fact that τ and μ are multiplicative, show that

$$\sum_{d|n} |\mu(d)|\tau(d) = 3^{\omega(n)}$$