

Number theory, Talteori 6hp, Kurskod TATA54, Provkod TEN1
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Each problem is worth 3 points. To receive full points, a solution needs to be complete. Indicate which theorems from the textbook that you have used, and include all auxiliary calculations.

No aids, no calculators, tables, nor textbooks.

- 1) Determine all solutions to $180x \equiv 120 \pmod{240}$.
- 2) Find all solutions to the congruence

$$x^7 + x^3 + x + 1 \equiv 0 \pmod{16}.$$

- 3) Consider the polynomial $f(t) = t^4 + 2t^2 - 4$. Does f have a zero which is an integer? A zero mod 19? A zero mod 43? Find examples of such zeroes, when possible.
- 4) Write 41 as a sum of two squares, and then write 205 as a sum of two squares. Finally, write 222 as a sum of four squares.
- 5) Find the continued fraction expansion of $\sqrt{17}$, then approximate $\sqrt{17}$ with a rational number, with an error less than 0.002.
- 6) Let f be a multiplicative arithmetical function. If the argument n has prime factorization $n = p_1^{a_1} \cdots p_k^{a_k}$, show that

$$\sum_{d|n} \mu(d)f(d) = (1 - f(p_1)) \cdots (1 - f(p_k)).$$

Use this to show that

$$\sum_{d|n} \frac{\mu(d)}{d} = \frac{\phi(n)}{n}.$$

- 7) Determine all positive integer solutions to $x^2 + 2y^2 = z^2$.