Number theory, Talteori 6hp, Kurskod TATA54, Provkod TEN1
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The first problem is worth 4 points, the second problem 2; the rest are all worth 3 points. To receive full points, a solution needs to be complete. Indicate which theorems from the textbook that you have used, and include all auxillary calculations.

No aids, no calculators, tables, nor textbooks.

8-10p: grade 3, 11-13p: grade 4, 14-18p grade 5.

1) Determine all solutions to the congruence

$$f(x) \equiv 0 \mod 2^k$$

for $1 \leq k \leq 3$, when

(a) $f(x) = x^2 + x$,

(b) $f(x) = 2x^2$.

2) Calculate

$$\alpha = 1 + \cfrac{1}{2 + \cfrac{1}{1 + \cfrac{1}{2 + \cfrac{1}{1 + \ldots}}}}$$

3) Let $n = 20000128$. Determine the positive integer k such that 2^k divides n but 2^{k+1} does not divide n.

4) Show that all sufficiently large integers can be expressed as a non-negative integer combination of 9 and 11, and determine the largest integer that can not be so expressed.

5) For which primes p is the congruence

$$x^2 \equiv 5 \mod p$$

solvable?

6) Find a positive integer a which is a primitive root modulo 5^k for all integers $k \geq 1$.