

Kurskod: TATA 54

Provkod: TEN 1

NUMBER THEORY, Talteori 6 hp

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Inga hjälpmedel är tillåtna! (For example books or pocket calculators are not allowed!)

You may write in Swedish, if you do this consistently.

You are rewarded at most 3 points for each of the 6 problems.

To get grade 3, 4 or 5, you need respectively 7, 11 and 14 points.

- (1) (a) How many (incongruent) primitive roots does the prime number 113 have ?
(b) Show that 2 is not a primitive root modulo 113.
- (2) Factorize the Gaussian integer $11 - 8i$ into Gaussian primes !
- (3) Factorize the number $5^{12} - 1$ into prime numbers !
- (4) (a) Compute the Jacobi symbol

$$\left(\frac{28}{143} \right)$$

- (b) Decide whether the congruence $x^2 \equiv 28 \pmod{143}$ is solvable or not !
- (5) (a) Show that the diophantine equation $x^2 - 7y^2 = -1$ has no solutions !
(b) Let n be a positive integer, such that $p|n$ for some prime number p with $p \equiv 3 \pmod{4}$. Show that the diophantine equation $x^2 - ny^2 = -1$ has no solutions !
- (6) (a) Show that

$$n \sum_{d|n} d^{p-2} \equiv \sigma(n) \pmod{p}$$

for each prime number p , which is not a prime divisor of n . Here $\sigma(n)$ is the sum of the divisors function.

- (b) Show that if n is a perfect number and p is a prime not dividing n , then

$$\sum_{d|n} d^{p-2} \equiv 2 \pmod{p}$$