Make-up exercises for TATA55, group theory part, 2019

November 22, 2019

1. Let G be an abelian group, and suppose that $x, y \in G$ both have order 2. Show that $o(xy) \leq 2$. When is the order exactly 2?

2. If a group has order 4, show that it is either cyclic or isomorphic to Kleins Viergruppe, (which is isomorphic $C(2) \times C(2)$).

3. Give an example of a non-abelian group G and elements $x, y \in G$, both having order 2, and where xy does not have order 2. (Hint: xy can have infinite order. You can find examples in the group of invertible matrices.)