## Exercises for TATA55, batch 1, 2023

August 28, 2023

Solutions to the exercises below should be handed in no later than september XXX, 2023. You may use a calculator. When proving statements you may freely use results that precedes the statement "in the logical development of the theory", but not results that come later.

1. (4p) Find all solutions to

$$
2 x+3 y+5 z=2001, \quad x, y, z \in \mathbb{Z}
$$

Which solutions have $x, y$, and $z$ positive?
2. (4p) Solve

$$
\begin{array}{ll}
x \equiv 2 & \bmod 13 \\
x \equiv 3 & \bmod 17 \\
x \equiv 4 & \bmod 19
\end{array}
$$

3. (4p) Show the following induction principle: suppose that $S \subseteq \mathbb{N} \times \mathbb{N}$ satisfies
(a) $(0,0) \in S$
(b) If $(a, b) \in S$ then $(a+1, b) \in S$ and $(a, b+1) \in S$

Then $\mathrm{S}=\mathbb{N} \times \mathbb{N}$.
4. (4p) Suppose that $\operatorname{gcd}(a, n)=d$, and that $d \mid b$. Show that the congruence $\mathrm{ax} \equiv \mathrm{b} \bmod \mathrm{n}$ is equivalent to the congruence $(\mathrm{a} / \mathrm{d}) \mathrm{x} \equiv \mathrm{b} / \mathrm{d} \bmod \mathrm{n} / \mathrm{d}$. How many different solutions $(\bmod n)$ does this congruence have? Find all non-congruent solutions to the congruence $10 x \equiv 35 \bmod 25$.

