Monomial ideals in 2 variables

$R = \mathbb{Q}[x,y] = \mathbb{Q}[M]$, $\quad M = \{x^a y^b : a,b \in \mathbb{N}\}$

$\mathbb{N}^2 \longrightarrow M$

$(a,b) \longmapsto x^a y^b$ monoid iso

I monomial ideal $\iff$ I gen by monomials (elems in M)

$\iff$ If $f = \sum_{m \in M} c_m m \in I$ then $m \in I$ all $m \in \operatorname{Supp}(f)$

$\iff$ I as $\mathbb{Q}$-v.s has basis of monomials

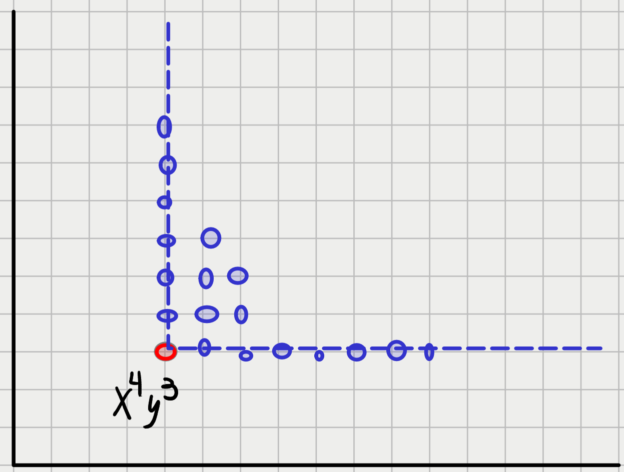
If I monomial ideal then

$J = I \cap M$ monoid ideal in M,

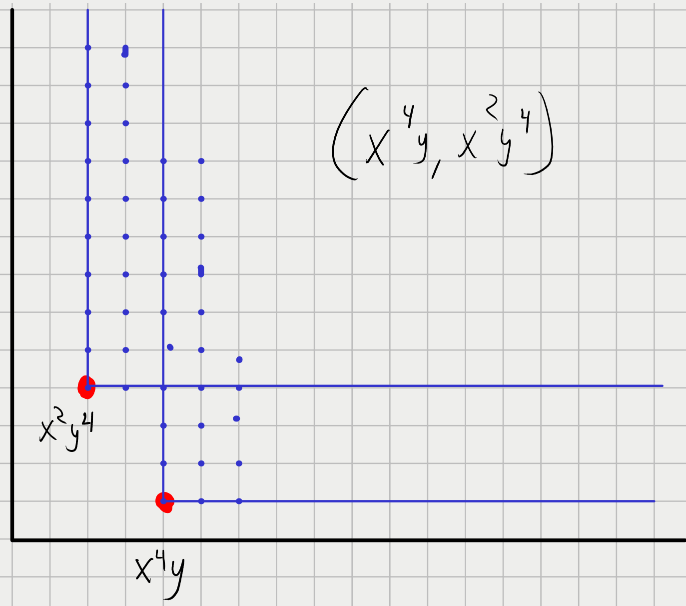
$I = \mathrm{Span}_{\mathbb{Q}}(J)$.

<u>Ex 1</u> Principal monomial ideal $I = (m)$

$m = x^a y^b$, $J = \{ x^c y^d : c \geq a,\ d \geq b \}$

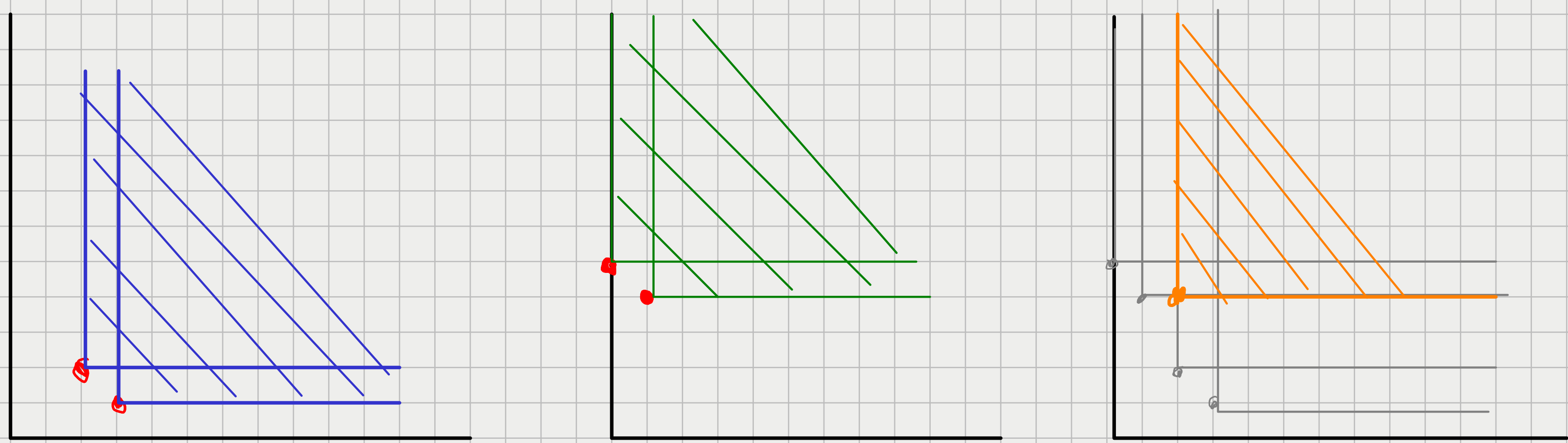


Ex 2 $(m_1) + (m_2) = (m_1, m_2)$



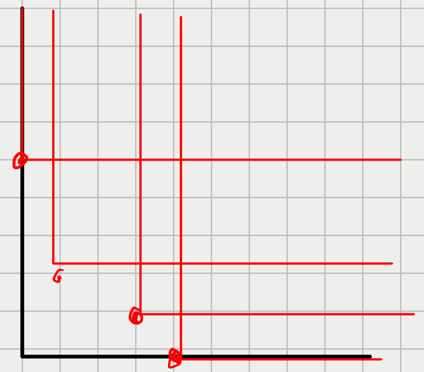
(x^4y, x^2y^4)

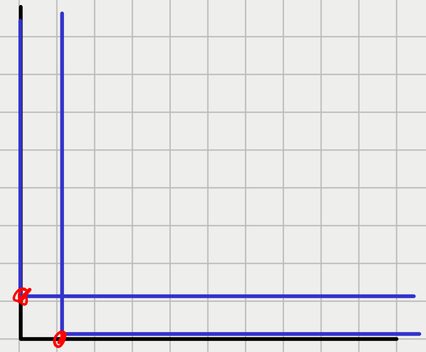
Ex 3 $I \cap J$

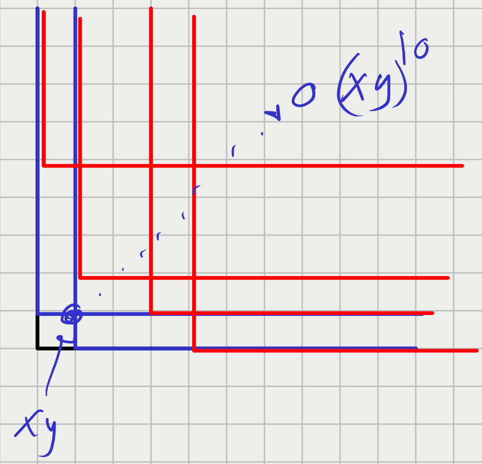


$(x^3y, x^2y^2) \cap (y^5, xy^4) = (x^2y^4)$

Radicals: $\sqrt{(x^4, x^3y, xy^2, y^5)} = (x, y)$







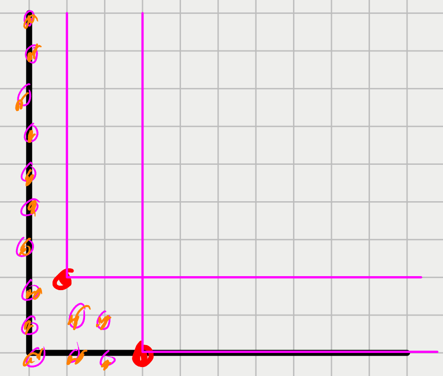
Prime ideals: (x), (y), (x, y)

Primary ideals: (x^9), (y^6), (x^9, y^6)

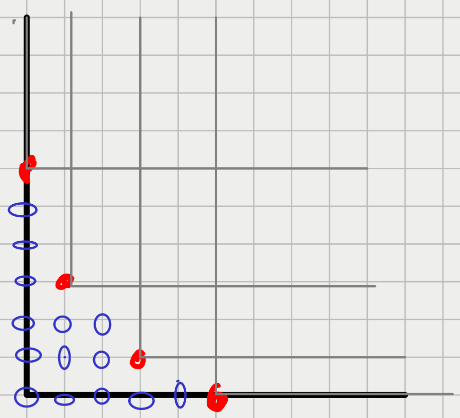
$\mathbb{Q}[x,y]$ has $\mathbb{Q}$-basis of monomials

$I \cap M$ $\mathbb{Q}$-basis for I as $\mathbb{Q}$-v.s

$M \setminus (I \cap M)$ $\mathbb{Q}$-basis for $\dfrac{\mathbb{Q}[x,y]}{I}$



$\mathbb{Q}$-Basis for $\dfrac{\mathbb{Q}[x,y]}{(x^3, xy^2)}$ is infinite



$\mathbb{Q}$-basis for $\dfrac{\mathbb{Q}[x,y]}{(x^5, x^3y, xy^3, y^6)}$

is $1, y, y^2, y^3, y^4, y^5, x, xy, xy^2, x^2, x^2y, x^2y^2, x^3, x^4$

Primary decomposition

$$I = (x^4, x^3y, y^4) = (x^4, y) \cap (x^3, y^4)$$