Exempel

Beräkna

$$\iiint_D \sin(x+y+z)\,dxdydz$$

där D är den parallellepiped som ges av olikheterna $0 \le x+y+z \le \pi/2$, $0 \le 2x+y+z \le 2$ och $0 \le x+y+3z \le 1$.

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$$dxdydz = \left| \frac{d(x,y,z)}{d(u,v,w)} \right| dudvdw = \left| \frac{d(u,v,w)}{d(x,y,z)} \right|^{-1} dudvdw = \frac{1}{2} dudvdw.$$

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$$\iiint_D \sin(x+y+z)\,dxdydz =$$
$$\int_0^1 \left(\int_0^2 \left(\int_0^{\pi/2} \sin u \frac{1}{2} du \right) dv \right) dw$$

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$$\frac{1}{2} \int_0^1 dw \cdot \int_0^2 dv \cdot \int_0^{\pi/2} \sin u du = \ldots = 1.$$