Exempel

Beräkna

$$\iiint_D \frac{1}{\sqrt{z}\sqrt{x^2+y^2}}dxdydz,$$

där $D = \{(x,y,z) \in \mathbb{R}^3 : 0 < z < 1 - x^2 - y^2\}$.

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Om vi inför polära koordinater i xy-planet ges området av

$$1/n \leq z < 1 - \rho^2, \quad 0 \leq \varphi < 2\pi, \quad 1/n \leq \rho \leq \sqrt{1 - 1/n}.$$

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$$\iiint_D \frac{1}{\sqrt{z}\sqrt{x^2+y^2}} dxdydz =$$

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$$\lim_{n\to\infty} 4\pi \int_{1/n}^{\sqrt{1-1/n}} (\sqrt{1-\rho^2} - \sqrt{1/n}) d\rho =$$

$$4\pi \int_0^1 \sqrt{1-\rho^2} d\rho = 4\pi \int_0^{\pi/2} \sqrt{1-\sin^2 t} \cos t dt$$

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