"Teori-Exempel"

Beräkna

$$f'_x = \frac{\partial f}{\partial x} \quad \text{och} \quad f'_y = \frac{\partial f}{\partial y}$$

då

$$f(x, y) = \sqrt{1 - x^2 - 2y^2}.$$

Bestäm speciellt $f'_x(0.7, 0)$ och $f'_y(0.7, 0)$, samt ange en ekvation för tangentplanet till funktionsytan i punkten som svarar mot $(x, y) = (0.7, 0)$.

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Geometrisk tolkning av $f'_x(0.7, 0)$

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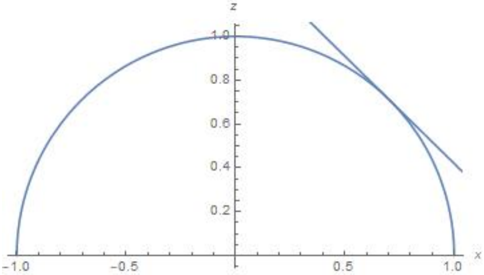
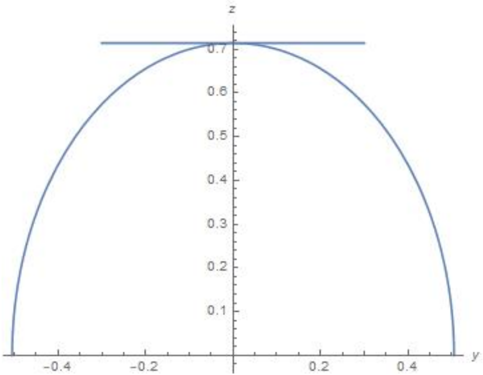
$f'_x(0.7, 0)$ är derivatan av funktionen $f(x, 0) = \sqrt{1 - x^2}$ i $x = 0.7$:



Geometrisk tolkning av $f'_y(0.7, 0)$

Geometrisk tolkning av $f_y'(0.7, 0)$

$f_y'(0.7, 0)$ är derivatan av funktionen $f(0.7, y) = \sqrt{0.51 - 2y^2}$ i $y = 0$:



Tangentplan

Vi får nu att

$$f(0.7+h, 0+k) \approx f(0.7, 0)+f_x'(0.7, 0)h+f_y'(0.7, 0)k = \sqrt{0.51}-\frac{0.7h}{\sqrt{0.51}}.$$

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