Exempel

Bestäm z''_{yy} uttryckta i derivator med avseende på u och v då

$$\begin{cases} u = x + 3y, \\ v = 2x - y. \end{cases}$$

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$$\frac{\partial^2 z}{\partial y^2}$$

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