TATA69 Flervariabelanalys

Föreläsning 9: Dubbelintegraler

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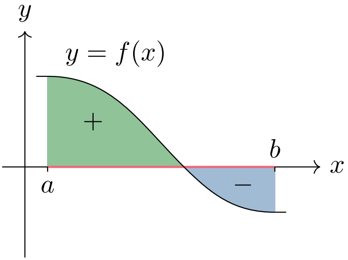
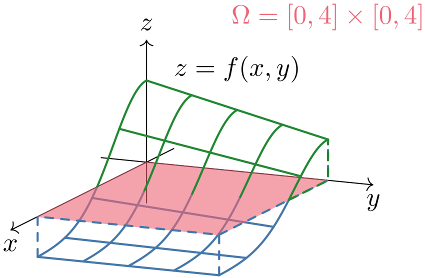
Integraler

Repetition och dubbelintegraler

Area under grafen med tecken $= \int_a^b f(x)dx$

Volumen under grafen med tecken $= \iint_\Omega f(x,y)dxdy$



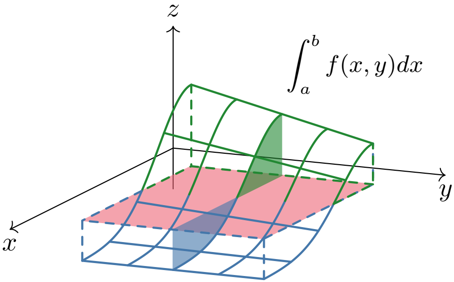


Sats 9.2

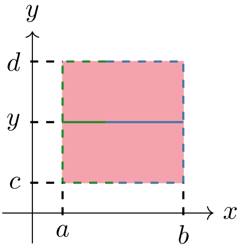
- $\iint_\Omega af(x,y) + bg(x,y)dxdy = a\iint_\Omega f(x,y)dxdy + b\iint_\Omega g(x,y)dxdy$
- $\iint_{\Omega_1\cup\Omega_2} f(x,y)dxdy = \iint_{\Omega_1} f(x,y)dxdy + \iint_{\Omega_2} f(x,y)dxdy$ där $\Omega_1 \cap \Omega_2 = \emptyset$
- $f \leq g \Rightarrow \iint_\Omega f(x,y)dxdy \leq \iint_\Omega g(x,y)dxdy$

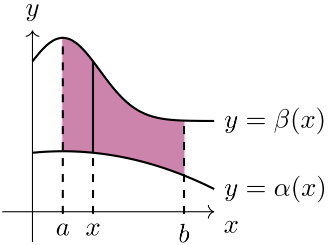
Fubinis sats

$$\iint_{\Omega} f(x,y)dxdy = \int_c^d \left(\int_a^b f(x,y)dx \right) dy$$



$$\Omega = [a,b] \times [c,d]$$

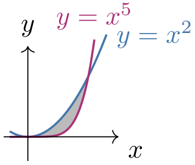




$$\Omega = \{(x,y)\colon \alpha(x) \leq y \leq \beta(x), a \leq x \leq b\}$$

$$\iint_{\Omega} f(x,y)dxdy = \int_a^b \left(\int_{\alpha(x)}^{\beta(x)} f(x,y)dy \right) dx$$

Exempel

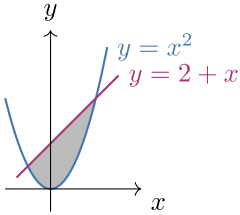


Exempel: Beräkna $\iint_D 6x^2y dx dy$ där $D \subseteq \mathbf{R}^2$ är det begränsade området som innesluts av kurvorna $y = x^2$ och $y = x^5$.

Lösning: $\iint_D 6x^2y dx dy = \int_0^1 \left(\int_{x^5}^{x^2} 6x^2y dy \right) dx =$

$\int_0^1 \left[3x^2y^2\right]_{x^5}^{x^2} dx \; = \; \int_0^1 3x^6 \; - \; 3x^{12} dx \; =$

$\left[\frac{3x^7}{7} - \frac{3x^{13}}{13}\right]_0^1 = \frac{3}{7} - \frac{3}{13} = \frac{3 \times (13-7)}{91} = \frac{18}{91}$



Exempel: Bestäm $\iint_M e^x dx dy$ där $M \subseteq \mathbf{R}^2$ är det begränsade området som innesluts av kurvorna $y = 2 + x$ och $y = x^2$.

Lösning: Skärningspunkter: $2 + x = x^2 \iff 0 = x^2 - x - 2 \iff 0 = (x+1)(x-2) \iff x \in \{-1, 2\}$.

$$\iint_M e^x dx dy \; = \; \int_{-1}^2 \left(\int_{x^2}^{2+x} e^x dy \right) dx \; = \; \int_{-1}^2 \left[ye^x\right]_{x^2}^{2+x} dx \; =$$

$$\int_{-1}^2 \left(2 + x - x^2\right) e^x dx = \cdots = \left[(-1 + 3x - x^2)e^x\right]_{-1}^2 = e^2 + \frac{5}{e}.$$