

1. a) Se bok.

$$\begin{aligned} b) \int \frac{dx}{1+\sin x} &= \int \frac{t=\tan \frac{x}{2}}{\dots} = \int \frac{2dt/(1+t^2)}{1+2t/(1+t^2)} = \int \frac{2dt}{1+t^2+2t} = \\ &= \int \frac{2dt}{(1+t)^2} = -\frac{2}{1+t} + C = -\frac{2}{1+\tan \frac{x}{2}} + C. \end{aligned}$$

2. a) Se bok.

b) Låt $\varepsilon > 0$. Sätt $x=1+h$, $y=1+k$, $\rho=\sqrt{h^2+k^2}$. Om $xy \neq 4$:

$$\begin{aligned} \left| \frac{3y}{4-xy} - 1 \right| &= \left| \frac{3+3k-4+1+h+k+hk}{4-1-h-k-hk} \right| \leq \frac{|h|+4|k|+|h||k|}{|3-h-k-hk|} \leq \\ &\leq \left/ \begin{array}{l} \text{om } \rho \leq 1/2 \\ (3-\frac{1}{2}-\frac{1}{2}-\frac{1}{4} \geq \frac{7}{4}) \end{array} \right/ \leq \frac{\rho+4\rho+\frac{1}{2}\rho}{7/4} < \left/ \text{om } \rho < \frac{7\varepsilon}{22} \right/ < \varepsilon. \end{aligned}$$

Så om $|(x,y)-(1,1)| < \delta = \min(1/2, 7\varepsilon/22)$ är $\left| \frac{3y}{4-xy} - 1 \right| < \varepsilon$.

3. a) Se bok.

$$b) f(0+h, 0+k) = 0 + 0h + 0k + \sqrt{h^2+k^2} \cdot \frac{\sqrt{|h|^3+|k|^3}}{\sqrt{h^2+k^2}} \quad (\text{då } (h,k) \neq (0,0)).$$

$$\text{Sätt } \rho(h,k) = \frac{\sqrt{|h|^3+|k|^3}}{\sqrt{h^2+k^2}}, \quad (h,k) \neq (0,0). \quad |\rho(h,k)| = \sqrt{\frac{|h|^3+|k|^3}{h^2+k^2}} \leq$$

$$\leq \sqrt{\frac{2(h^2+k^2)^{3/2}}{h^2+k^2}} = \sqrt{2}(h^2+k^2)^{1/4} \rightarrow 0, \quad (h,k) \rightarrow (0,0), \quad \text{så } f \text{ är}$$

differentierbar i $(0,0)$.

4. a) Se bok.

$$\begin{aligned} b) f(0,0) &= 0. \quad f(t,t^5) = (t^5+t^4)(t^5+t^6) = t^4(t+1)t^5(1+t) = \\ &= t^9(t+1)^2 < 0 \quad \text{då } -1 < t < 0, \quad \text{så } f \text{ har inte lok. min. i } (0,0). \end{aligned}$$

5. a) Se bok.

$$\begin{aligned} b) \int_0^1 \left(\int_0^{\arccos y} e^{\sin x} dx \right) dy &= \left/ \begin{array}{c} y \\ 1 \\ \pi/2 \end{array} \right/ = \int_0^{\pi/2} \left(\int_0^{\cos x} e^{\sin x} dy \right) dx = \\ &= \int_0^{\pi/2} \cos x \cdot e^{\sin x} dx = \left[e^{\sin x} \right]_0^{\pi/2} = \underline{\underline{e-1}}. \end{aligned}$$

6. Se bok.