

F14

Ex. Consider the recurrence $a_{n+2} = 5a_{n+1} - 6a_n + 3^{n+1}$, $n \in \mathbb{N}$.

(a) Show that the solution which satisfies $a_0 = 0$, $a_1 = 3$ is $a_n = n \cdot 3^n$.

(b) Find the solution which satisfies $a_0 = 5$, $a_1 = 17$.

Ex. Let $\mathcal{F}$ be any set whose elements are sets.

(a) Show that $\subseteq$ is a partial order on $\mathcal{F}$.

(b) Now, $\mathcal{F} = \{S \subseteq \{1, \dots, n\} : |S| \text{ even}\}$.

For which $n \in \mathbb{Z}_+$ is $(\mathcal{F}, \subseteq)$ a lattice?

Ex. Reduce $131^{40} \pmod{273}$. (Hint: $273 = 3 \cdot 7 \cdot 13$.)

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Ex. How many $(0/1)$ -matrices of size 4×4 have

(a) no all-zero row?

(b) no all-zero row or column?

Ex. ($\approx 2021.10.22.6$) In how many ways can one choose a 4-subset of $\{1, \dots, 100\}$ so that not all four are odd, not all divisible by 3, and not all divisible by 5?

Ex. Suppose G is a graph with minimum vertex degree ≥ 3 .

Show that G contains a cycle of length ≥ 4 .

(Hint: consider a path $v_0, \dots, v_m$ of max. length. Where are the neighbours of v_0 ?)

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Ex. (cont'd) Suppose G is a graph without cycles of length ≥ 4 .
Prove: $\chi(G) \leq 3$