F2

- Recursively defined sequences

ex. Fibonacci: $F_0 = 0,\ F_1 = 1$

ex. $a_n = \#\text{subsets of } \{1,\dots,n\} \rightsquigarrow a_n = 2a_{n-1}\ \ \forall n \in \mathbb{Z}_+ \ (a_0 = 1)$

$(\rightsquigarrow a_n = 2^n)$

Ex. Define a_n by $a_0 = 0,\ a_1 = -2$ and $a_n = -4a_{n-1} - 4a_{n-2} - (-2)^{n+1}$ if $n \geq 2$.

Prove that $a_n = n^2 \cdot (-2)^n \ \ \forall n \in \mathbb{N}$.

- Linear recurrence eq. with constant coefficients

$$a_{n+k} + c_{k-1}a_{n+k-1} + c_{k-2}a_{n+k-2} + \dots + c_0 a_n \overset{(*)}{=} f(n)$$

char. poly. $x^k + c_{k-1}x^{k-1} + \dots + c_0$

- Thm (*) hom. $(f(n) = 0)$, general solution.

if time [Pf (case of simple roots)]

Ex. Solve (Fib.) $a_{n+2} = a_{n+1} + a_n$, $n \in \mathbb{N}$

if time [Init. cond.]

- Thm (*) inhom.: $a_n = a_n^{hom} + a_n^{part}$ if time [Pf]
- If $f(n) = (\text{poly}) \cdot \alpha^n$, ansatz (multiply by n if necessary to get away from a_n^{hom})

Ex. Solve $a_n = -4a_{n-1} - 4a_{n-2} - (-2)^{n+1}$

(without using above info)

if time [with $a_0 = 0,\ a_1 = -2$]