

F3

$A \cup B = |A| + |B|, |A \times B| = |A| \cdot |B|$

→ sum rule, product rule

• Principle: # stuff = #ways to construct stuff

Ex. # subsets of  $\{1, \dots, n\}$   
# functions  $\{1, \dots, n\} \rightarrow \{1, \dots, 7\}$ .

• Ordered selection: "k-permutation" of n-set  
permutation = "n-perm."

$P(n, k) \stackrel{\text{def}}{=} \# \dots$

Obs.  $P(n, k) = n(n-1) \dots (n-k+1) \quad (k \geq 1)$   
( $P(n, n) = n!$ )  
 $0! = 1$ .

Ex. # injective functions  $f: \{1, \dots, n\} \rightarrow \{1, \dots, 7\}$

• Unordered selection: "k-combination" = k-subset of n-set.

$\binom{n}{k} \stackrel{\text{def}}{=} \# \dots \quad (n \in \mathbb{N}, k \in \mathbb{Z})$

Prop.  $\binom{n}{k} = \begin{cases} 0 & \text{if } k < 0 \text{ or } k > n, \\ \frac{n(n-1) \dots (n-k+1)}{k!} & \text{if } 0 \leq k \leq n. \end{cases}$  ( $k=0$ : empty prod. = 1)  
Pf.

Ex. A group of 10 women, 8 men elect a committee of  
1 chair, 1 co-chair, 1 secretary, 4 other members.

If chair is male, both co-chair and secretary must be female,  
and vice versa. Among the 4, both genders must be rep.

In how many ways...?

Ex. How many 4-letter "words" can be formed using  
letters from CHICKADEE?

if time  $\left[ \begin{array}{l} \bullet \text{ Binomial th. + pf.} \end{array} \right.$

Ex. Prove  $\sum_{k=0}^n \binom{n}{k} = 2^n$  in two ways.