(F6)

- Divisor, divisibility ("almost $\frac{a}{b} \in \mathbb{Z}$")

Ex. Prove: $n \in \mathbb{Z}$ odd $\Rightarrow 8 \mid n^2-1$.

- gcd   Note: $\gcd(a,0) = a$ if $a>0$. ($\gcd(0,0) = ?$)
- prime

Thm. ∞ many primes. (Every int. ≥ 2 div. by prime)

- FTOA (w/o pf, at least now)

ex. $\gcd(234,42) = \dots = 6$

- lcm; $\operatorname{lcm}(a,b) = \frac{ab}{\gcd(a,b)}$ $(a,b \in \mathbb{Z}_+)$

ex. $\operatorname{lcm}(234,42)$

Ex. Find $|\{d \in \mathbb{Z}_+ : 60 \mid d \mid 5040\}|$.

Thm ("div. alg.") $a \in \mathbb{Z}, b \in \mathbb{Z}_+ \Rightarrow \exists!\, q, r \in \mathbb{Z}$ with $0 \leq r < b$
such that $a = qb + r$.

Prop. $\gcd(a,b) = \gcd(b,r)$
$\rightsquigarrow$ Euclid's alg.

ex. $\gcd(234,42)$ via Euclid.

Thm. (Bezout's id.) $\exists \alpha, \beta \in \mathbb{Z}: \alpha a + \beta b = \gcd(a,b)$

Ex. Find $\alpha, \beta \in \mathbb{Z}$ s/t $234\alpha + 42\beta = 6$

if time [Show: p prime, $p \mid ab \Rightarrow p \mid a$ or $p \mid b$ (Key to FTOA.)]

- Diophantine eq., $ax + by = c$
- observations:
  - $\gcd(a,b) \nmid c \Rightarrow$ no sol.
  - $\gcd(a,b) \mid c \Rightarrow$ sol. exists, find with Bezout
  - $x = x_0, y = y_0$ one sol. $\rightarrow \begin{cases} x = x_0 + \frac{b}{\gcd(a,b)} t \\ y = y_0 - \frac{a}{\gcd(a,b)} t \end{cases}, t \in \mathbb{Z}$.
  - WLOG $\gcd(a,b) = 1$, if desired.

Ex. Solve a) $234x + 42y = 12$
b) $ = 13$

if time [Ex. Solve $5x + 3y + 6z = 4$.]