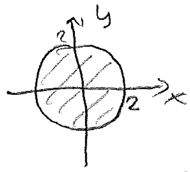
Tips och bilder 6.9ab, 6.10abc, 6.11, (6.13), 6.29

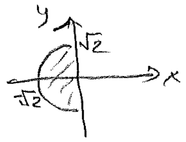
(därefter lösningen)

6.9a Polära koord $\begin{cases} x=\rho\cos\varphi \\ y=\rho\sin\varphi \end{cases}$



$\begin{cases} 0\le\rho\le 2 \\ 0\le\varphi<2\pi \end{cases}$ $\underbrace{\dfrac{d(x,y)}{d(\rho,\varphi)}=\rho\ (\ge 0)}_{\text{behöver man inte härleda varje gång}}$

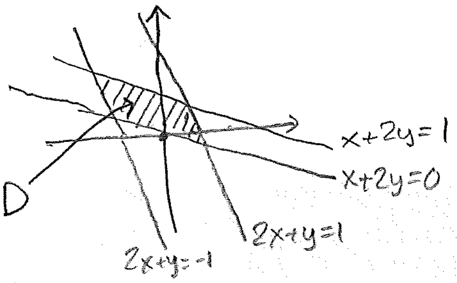
6.9b Polära $\begin{cases} x^2+y^2\le 2 \\ x\le 0 \end{cases}$



ger $\begin{cases} 0\le\rho\le\sqrt{2} \\ \frac{\pi}{2}\le\varphi\le\frac{3\pi}{2} \end{cases}$

6.10a $D:\begin{cases} 0\le x+2y\le 1 \\ -1\le 2x+y\le 1 \end{cases}$

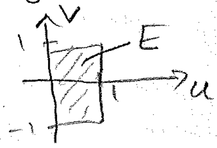
parallellogram



Nya variabler

$\begin{cases} u=x+2y \\ v=2x+y \end{cases}$ ger

$E:\begin{cases} 0\le u\le 1 \\ -1\le v\le 1 \end{cases}$



Linjärt variabelbyte ger konstant funktionaldeterminant

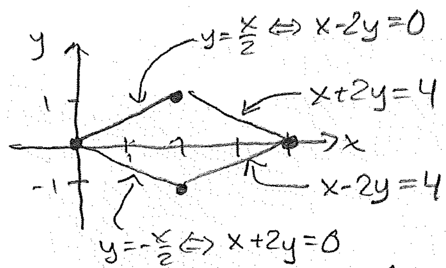
$$\frac{d(x,y)}{d(u,v)}=\frac{1}{\frac{d(u,v)}{d(x,y)}}=\frac{1}{\begin{vmatrix} u'_x & u'_y \\ v'_x & v'_y \end{vmatrix}}=\frac{1}{\begin{vmatrix} 1 & 2 \\ 2 & 1 \end{vmatrix}}=\frac{1}{-3}$$

(kom ihåg att ta belopp av detta vid integralberäkningen)

6.10b D: romb med hörn i (0,0), (2,1), (4,0), (2,-1)

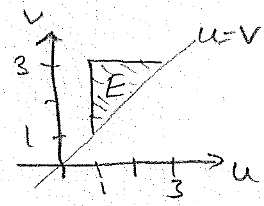
D kan skrivas $\begin{cases} 0\le x+2y\le 4 \\ 0\le x-2y\le 4 \end{cases}$

Gör variabelbytet $\begin{cases} u=x+2y \\ v=x-2y \end{cases}$ (linjärt byte)



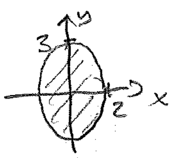
(tänk ut ekvationen för kantlinjen)

6.10c $D:\ 1\le \underbrace{x+y}_{u}\le \underbrace{2x-4y}_{v}\le 3$ ger $E\ \ 1\le u\le v\le 3$



(Bild i xy: Rita $x+y=1$, $\underbrace{x+y=2x-4y}_{x=5y}$, $2x-4y=3$ för att se triangel)

6.11a $\dfrac{x^2}{4}+\dfrac{y^2}{9}\le 1$ ellips



Sätt först $\begin{cases} u=\frac{x}{2} \\ v=\frac{y}{3} \end{cases}$ ger $E:\ u^2+v^2\le 1$

Därefter kan u, v bytas mot polära

2 steg, determinant kommer in i varje steg

[Alternativt: Direkt $\begin{cases} \frac{x}{2}=\rho\cos\varphi \\ \frac{y}{3}=\rho\sin\varphi \end{cases} \Leftrightarrow \begin{cases} x=2\rho\cos\varphi \\ y=3\rho\sin\varphi \end{cases}$

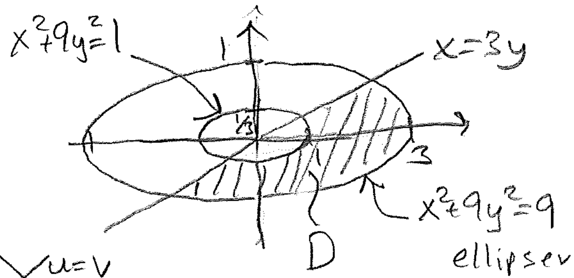
ger $0\le\rho\le 1,\ 0\le\varphi<2\pi$

en determinant (som måste beräknas, ej längre vanliga polära)]

Kom ihåg $\cos^2 t=\dfrac{1+\cos 2t}{2}$

$\sin^2 t=\dfrac{1-\cos 2t}{2}$

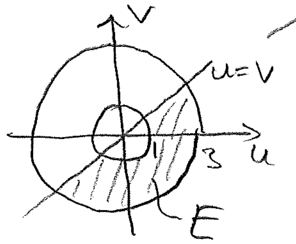
6.11 b $D: 1 \le x^2+9y^2 \le 9 \ , \ x \ge 3y$



D är ett halvt varv av området mellan två ellipser

$\begin{cases} u=x \\ v=3y \end{cases}$ gör om till cirklar

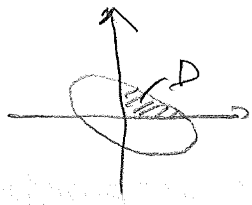
$E: 1 \le u^2+v^2 \le 9 \ , \ u \ge v$



Byt sedan u, v mot polära

6.11 c $D: \begin{cases} x^2+2xy+4y^2 \le 1 \\ x \ge 0 \\ y \ge 0 \end{cases}$ kvadratkomplettera $(x+y)^2+3y^2 \le 1$

någon sned ellips (grov skiss)

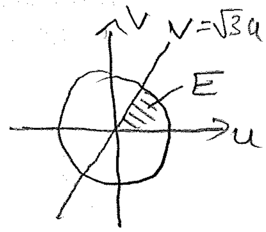


$u=x+y, v=\sqrt{3}y$ ger $u^2+v^2 \le 1$

Vad händer med $\begin{cases} x \ge 0 \\ y \ge 0 \end{cases}$. Obs inte samma som $\begin{cases} u \ge 0 \\ v \ge 0 \end{cases}$!

Lös ut $\Rightarrow \begin{cases} x = u-y = u - \frac{v}{\sqrt{3}} \ge 0 & \Rightarrow \underline{v \le \sqrt{3}u} \\ y = \frac{v}{\sqrt{3}} \ge 0 & \Rightarrow \underline{v \ge 0} \end{cases}$

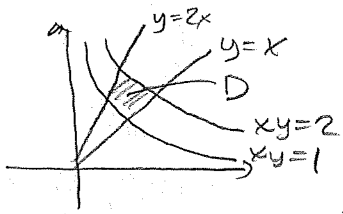
E blir $\begin{cases} u^2+v^2 \le 1 \\ v \le \sqrt{3}u \\ v \ge 0 \end{cases}$



Byt sedan u, v mot polära

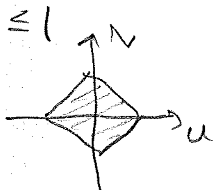
6.13 $D: 1 \le xy \le 2 \ , \ 0 < \underbrace{x \le y \le 2x}_{\text{dela med } x}$ $1 \le \frac{y}{x} \le 2$

Sätt $\begin{cases} u = xy \\ v = \frac{y}{x} \end{cases}$ $(\Rightarrow uv = y^2)$ ger $E: \begin{cases} 1 \le u \le 2 \\ 1 \le v \le 2 \end{cases}$



6.29 a) Def. av area $A(D) = \iint_D 1 \, dx dy$ $\quad D: |x+2y|+|3x-y| \le 1$

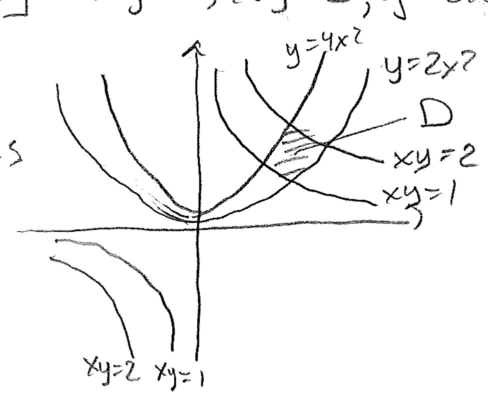
Sätt $u=x+2y, v=3x-y$ $\Rightarrow \underbrace{|u|+|v| \le 1}_{E}$ (som övn 1.2)



Konstant determinant $\Rightarrow A(D) = \iint_E \underbrace{\left|\frac{d(x,y)}{d(u,v)}\right|}\, du dv = \left|\frac{d(x,y)}{d(u,v)}\right| \cdot \underbrace{A(E)}_{\text{enkel utan integral!}}$

6.29 b) $xy=1, xy=2, y=2x^2, y=4x^2$

Skiss



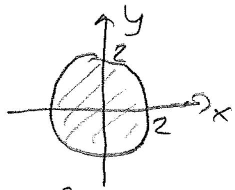
D ges av $\begin{cases} 1 \le xy \le 2 \\ 2x^2 \le y \le 4x^2 \Leftrightarrow 2 \le \frac{y}{x^2} \le 4 \end{cases}$

Sätt $u = xy \ , \ v = \frac{y}{x^2}$

Ej konstant determinant i detta fall

Lösningen

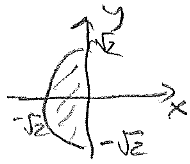
6.9 a) $I=\iint_D e^{x^2+y^2}\,dx\,dy$, polära koord ger $\begin{cases}0\le\rho\le 2\\ 0\le\varphi<2\pi\end{cases}$



$\frac{d(x,y)}{d(\rho,\varphi)}=\rho$, $x^2+y^2=\rho^2$

$$\Rightarrow I=\int_0^{2\pi}\int_0^2 e^{\rho^2}\underbrace{\left|\frac{d(x,y)}{d(\rho,\varphi)}\right|}_{=\rho}d\rho\,d\varphi=\int_0^{2\pi}\left[\tfrac{1}{2}e^{\rho^2}\right]_0^2 d\varphi=\tfrac{1}{2}\int_0^{2\pi}(e^4-e^0)\,d\varphi=\frac{e^4-1}{2}\int_0^{2\pi}d\varphi=\frac{e^4-1}{2}\left[\varphi\right]_0^{2\pi}=$$
$$=\pi(e^4-1)$$

6.9 b) $I=\iint_D \frac{x\,dx\,dy}{1+(x^2+y^2)^{3/2}}$ $\quad D:\begin{cases}x^2+y^2\le 2\\ x\le 0\end{cases}$



polära ger $\begin{cases}0\le\rho\le\sqrt{2}\\ \frac{\pi}{2}\le\varphi\le\frac{3\pi}{2}\end{cases}$

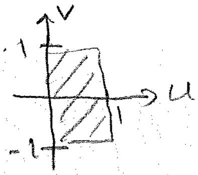
$$\Rightarrow I=\int_{\pi/2}^{3\pi/2}\int_0^{\sqrt{2}}\frac{\rho\cos\varphi}{1+(\rho^2)^{3/2}}\,\rho\,d\rho\,d\varphi=\int_{\pi/2}^{3\pi/2}\left(\int_0^{\sqrt{2}}\frac{\rho^2}{1+\rho^3}d\rho\right)\cos\varphi\,d\varphi=\int_{\pi/2}^{3\pi/2}\left[\tfrac{1}{3}\ln(1+\rho^3)\right]_0^{\sqrt{2}}\cos\varphi\,d\varphi=$$

determinanten

sätt $t=\rho^3$ eller $t=1+\rho^3$ om ni inte "ser" inversderivata

$$=\tfrac{1}{3}\int_{\pi/2}^{3\pi/2}\left(\ln(1+2\sqrt{2})-\underbrace{\ln 1}_{=0}\right)\cos\varphi\,d\varphi=\frac{\ln(1+2\sqrt{2})}{3}\underbrace{\left[\sin\varphi\right]_{\pi/2}^{3\pi/2}}_{=-1-1}=-\tfrac{2}{3}\ln(1+2\sqrt{2})$$

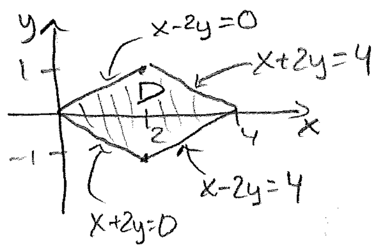
6.10 a) $D:\begin{cases}0\le x+2y\le 1\\ -1\le 2x+y\le 1\end{cases}$ $\quad\begin{cases}u=x+2y\\ v=2x+y\end{cases}$ ger $\left.\begin{matrix}0\le u\le 1\\ -1\le v\le 1\end{matrix}\right\}E$



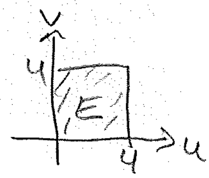
$$\frac{d(x,y)}{d(u,v)}=\frac{1}{\frac{d(u,v)}{d(x,y)}}=\frac{1}{\begin{vmatrix}1&2\\2&1\end{vmatrix}}=-\frac{1}{3}$$

$$I=\iint_D(x+2y)\cos(2x+y)\,dx\,dy=\iint_E u\cos v\underbrace{\left|\frac{d(x,y)}{d(u,v)}\right|}_{=1/3}du\,dv=\tfrac{1}{3}\int_{-1}^{1}\left(\int_0^1 u\,du\right)\cos v\,dv=$$
$$=\tfrac{1}{3}\int_{-1}^{1}\underbrace{\left[\frac{u^2}{2}\right]_0^1}_{=1/2}\cos v\,dv=\tfrac{1}{6}\left[\sin v\right]_{-1}^{1}=\tfrac{1}{6}\big(\sin 1-\underbrace{\sin(-1)}_{-\sin 1}\big)=\frac{\sin 1}{3}$$

6.10 b)



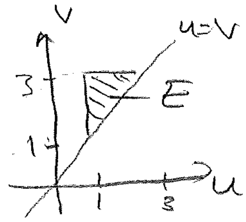
$\begin{cases}u=x+2y\\ v=x-2y\end{cases}$ ger $E:\begin{cases}0\le u\le 4\\ 0\le v\le 4\end{cases}$



$$\frac{d(x,y)}{d(u,v)}=\frac{1}{\frac{d(u,v)}{d(x,y)}}=\frac{1}{\begin{vmatrix}1&2\\1&-2\end{vmatrix}}=-\frac{1}{4}$$

$$I=\iint_D(x+2y)e^{x-2y}\,dx\,dy=\iint_E u e^v\left|-\tfrac{1}{4}\right|du\,dv=\tfrac{1}{4}\int_0^4\underbrace{\left[\frac{u^2}{2}\right]_0^4}_{8-0}e^v\,dv=2\left[e^v\right]_0^4=$$
$$=2(e^4-1)$$

6.10c] $D: 1 \le x+y \le 2x-4y \le 3$ $\quad \begin{cases} u = x+y \\ v = 2x-4y \end{cases}$ ger $E: 1 \le u \le v \le 3$



E kan skrivas $\begin{cases} 1 \le u \le 3 \\ u \le v \le 3 \end{cases}$ eller $\begin{cases} 1 \le v \le 3 \\ 1 \le u \le v \end{cases}$

$$\frac{d(x,y)}{d(u,v)} = \frac{1}{\frac{d(u,v)}{d(x,y)}} = \frac{1}{\begin{vmatrix} 1 & 1 \\ 2 & -4 \end{vmatrix}} = -\frac{1}{6}$$, Uttryck $x-y+1$ i u och v:

$$\begin{cases} u = x+y \\ v = 2x-4y \end{cases} \quad \left(\begin{array}{cc|c} 1 & 1 & u \\ 2 & -4 & v \end{array}\right)_{-2(1)} \sim \left(\begin{array}{cc|c} 1 & 1 & u \\ 0 & -6 & v-2u \end{array}\right) \sim \left(\begin{array}{cc|c} 1 & 1 & u \\ 0 & 1 & \frac{2u-v}{6} \end{array}\right) \sim \left(\begin{array}{cc|c} 1 & 0 & u-\frac{2u-v}{6} = \frac{4u+v}{6} \\ 0 & 1 & \frac{2u-v}{6} \end{array}\right) \Rightarrow$$

$$\begin{cases} x = \frac{4u+v}{6} \\ y = \frac{2u-v}{6} \end{cases} \Rightarrow x-y+1 = \frac{4u+v}{6} - \frac{2u-v}{6} + 1 = \frac{u+v}{3} + 1 \quad \left(\text{och } \frac{d(x,y)}{d(u,v)} = \begin{vmatrix} 4/6 & 1/6 \\ 2/6 & -1/6 \end{vmatrix} = -\frac{6}{36} = -\frac{1}{6}\right)$$

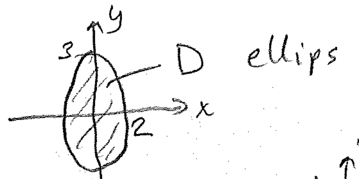
$$I = \iint_D \frac{x-y+1}{x+y}\,dxdy = \iint_E \frac{\frac{u+v}{3}+1}{u}\left|-\frac{1}{6}\right| dudv = \frac{1}{18}\iint_E \frac{u+v+3}{u}\,dudv \underset{\uparrow}{=} \frac{1}{18}\int_1^3 \int_u^3 \left(1+\frac{v}{u}+\frac{3}{u}\right) dv\,du$$

om vi integrerar v först

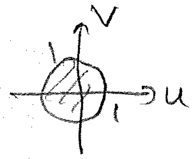
$$= \frac{1}{18}\int_1^3 \left[v + \frac{v^2}{2u} + \frac{3v}{u}\right]_{v=u}^{v=3} du = \frac{1}{18}\int_1^3 \left(3 + \frac{9}{2u} + \frac{9}{u} - u - \frac{u^2}{2u} - \frac{3u}{u}\right) du = \frac{1}{18}\int_1^3 \left(\frac{27}{2u} - \frac{3u}{2}\right) du =$$

$$= \frac{1}{18}\left[\frac{27}{2}\ln|u| - \frac{3u^2}{4}\right]_1^3 = \frac{1}{18}\left(\frac{27}{2}\ln 3 - \frac{27}{4} - 0 + \frac{3}{4}\right) = \underline{\frac{3}{4}\ln 3 - \frac{1}{3}}$$

6.11a] $\frac{x^2}{4} + \frac{y^2}{9} \le 1 \iff \left(\frac{x}{2}\right)^2 + \left(\frac{y}{3}\right)^2 \le 1$



$$I = \iint_D (x^2+y^2)\,dxdy \qquad \text{Sätt } \begin{cases} u = \frac{x}{2} \\ v = \frac{y}{3} \end{cases} \iff \begin{cases} x = 2u \\ y = 3v \end{cases} \text{ ger } E: u^2+v^2 \le 1$$



$$\frac{d(x,y)}{d(u,v)} = \begin{vmatrix} 2 & 0 \\ 0 & 3 \end{vmatrix} = 6$$

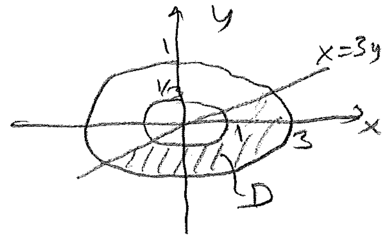
$$\Rightarrow I = \iint_E (4u^2+9v^2)\cdot|6|\,dudv =$$

$$= \Big/ \text{polära} \begin{cases} u = \rho\cos\varphi & 0 \le \rho \le 1 \\ v = \rho\sin\varphi & 0 \le \varphi < 2\pi \end{cases} \Big/ = 6\int_0^{2\pi}\int_0^1 (4\rho^2\cos^2\varphi + 9\rho^2\sin^2\varphi)\overset{\text{determinant}}{\overset{\downarrow}{\rho}}\,d\rho\,d\varphi =$$

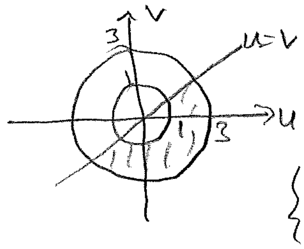
$$= 6\int_0^{2\pi}\int_0^1 \rho^3 \underbrace{(4\cos^2\varphi + 9\sin^2\varphi)}_{=4\cos^2\varphi+4\sin^2\varphi+5\cos^2\varphi = 4+5\cdot\frac{1-\cos 2\varphi}{2}} d\rho\,d\varphi = 6\int_0^{2\pi} \underbrace{\left[\frac{\rho^4}{4}\right]_0^1}_{=1/4} \left(4 + \frac{5}{2} - \frac{5}{2}\cos 2\varphi\right) d\varphi =$$

$$= \frac{6}{4}\left[\frac{13}{2}\varphi - \frac{5}{4}\sin 2\varphi\right]_0^{2\pi} = \frac{6}{4}\left(\frac{13}{2}\cdot 2\pi - 0 - 0 + 0\right) = \frac{39\pi}{2}$$

6.11b $I = \iint_D x^3 \, dx\,dy \quad D: 1 \le x^2+9y^2 \le 9,\ x \ge 3y$



$\begin{cases} u = x \\ v = 3y \end{cases}$ ger $E: 1 \le u^2+v^2 \le 9,\ u \ge v$



Polära $\begin{cases} u = \rho\cos\varphi \\ v = \rho\sin\varphi \end{cases}$ ger $\begin{cases} 1 \le \rho \le 3 \\ -\frac{3\pi}{4} \le \varphi \le \frac{\pi}{4} \end{cases}$

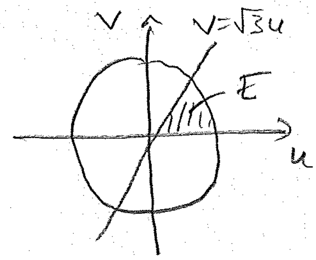
$\begin{cases} x = u \\ y = v/3 \end{cases} \Rightarrow \frac{d(x,y)}{d(u,v)} = \begin{vmatrix} 1 & 0 \\ 0 & 1/3 \end{vmatrix} = \frac{1}{3}$

$$I = \iint_E u^3 \cdot \left|\tfrac{1}{3}\right| du\,dv = \frac{1}{3}\int_{-3\pi/4}^{\pi/4}\int_1^3 \rho^3\cos^3\varphi \cdot \underset{\text{determinant}}{\rho}\, d\rho\, d\varphi = \frac{1}{3}\int_{-3\pi/4}^{\pi/4}\left[\frac{\rho^5}{5}\right]_1^3 \cdot \cos\varphi\overbrace{(1-\sin^2\varphi)}^{\cos^2\varphi}\,d\varphi =$$

$$= \frac{1}{15}\underbrace{(3^5-1)}_{242}\left[\sin\varphi - \frac{\sin^3\varphi}{3}\right]_{-3\pi/4}^{\pi/4} = \frac{242}{15}\left(\frac{1}{\sqrt{2}} - \frac{1}{6\sqrt{2}} + \frac{1}{\sqrt{2}} - \frac{1}{6\sqrt{2}}\right) = \frac{242}{15}\cdot\frac{5\sqrt{2}}{6} = \frac{121\sqrt{2}}{9}$$

6.11c $I = \iint_D x \, dx\,dy \quad D: \begin{cases} x^2+2xy+4y^2 \le 1 \\ x \ge 0 \\ y \ge 0 \end{cases} \Leftrightarrow \underbrace{(x+y)}_{u}^2 + \underbrace{(\sqrt{3}y)}_{v}^2 \le 1$

$\begin{cases} u = x+y \\ v = \sqrt{3}y \end{cases} \Leftrightarrow \begin{cases} x = u - \frac{v}{\sqrt{3}} \\ y = \frac{v}{\sqrt{3}} \end{cases}$ ger $E: \begin{cases} u^2+v^2 \le 1 \\ u \ge \frac{v}{\sqrt{3}} \\ v \ge 0 \end{cases}$



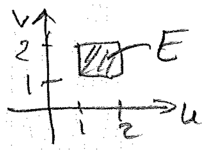
polära $\begin{cases} 0 \le \rho \le 1 \\ 0 \le \varphi \le \frac{\pi}{3} \end{cases}$

$\frac{d(x,y)}{d(u,v)} = \begin{vmatrix} 1 & -1/\sqrt{3} \\ 0 & 1/\sqrt{3} \end{vmatrix} = \frac{1}{\sqrt{3}} \quad \Rightarrow$

$$I = \iint_E \left(u - \frac{v}{\sqrt{3}}\right)\cdot\left|\frac{1}{\sqrt{3}}\right| du\,dv \underset{\text{polära}}{=} \frac{1}{\sqrt{3}}\int_0^{\pi/3}\int_0^1\left(\rho\cos\varphi - \frac{\rho\sin\varphi}{\sqrt{3}}\right)\rho\,d\rho\,d\varphi = \frac{1}{\sqrt{3}}\int_0^{\pi/3}\int_0^1 \rho^2\left(\cos\varphi - \frac{\sin\varphi}{\sqrt{3}}\right)d\rho\,d\varphi$$

$$= \frac{1}{\sqrt{3}}\int_0^{\pi/3}\underbrace{\left[\frac{\rho^3}{3}\right]_0^1}_{1/3}\left(\cos\varphi - \frac{\sin\varphi}{\sqrt{3}}\right)d\varphi = \frac{1}{3\sqrt{3}}\left[\sin\varphi + \frac{\cos\varphi}{\sqrt{3}}\right]_0^{\pi/3} = \frac{1}{3\sqrt{3}}\left(\frac{\sqrt{3}}{2} + \frac{1/2}{\sqrt{3}} - 0 - \frac{1}{\sqrt{3}}\right) = \frac{1}{9}\left(\frac{3}{2} + \frac{1}{2} - 1\right) = \frac{1}{9}$$

6.13 $1 \le \underbrace{xy}_{u} \le 2,\ 0 < \underline{x \le y \le 2x} \Rightarrow 1 \le \underbrace{\frac{y}{x}}_{v} \le 2$ $\quad \begin{cases} u = xy \\ v = y/x \end{cases} \Rightarrow E: \begin{cases} 1 \le u \le 2 \\ 1 \le v \le 2 \end{cases}$



$$\frac{d(x,y)}{d(u,v)} = \frac{1}{\frac{d(u,v)}{d(x,y)}} = \frac{1}{\begin{vmatrix} u'_x & u'_y \\ v'_x & v'_y \end{vmatrix}} = \frac{1}{\begin{vmatrix} y & x \\ -y/x^2 & 1/x \end{vmatrix}} = \frac{1}{\frac{y}{x} + \frac{y}{x}} = \frac{1}{2v}$$ Notera $uv = y^2 \Rightarrow$

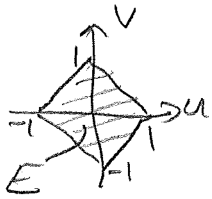
$$I = \iint_D y^2\sin(y^2)\,dx\,dy = \iint_E uv\sin(uv)\left|\frac{1}{2v}\right| du\,dv \underset{(v>0 \text{ i } E)}{=} \frac{1}{2}\iint_E u\sin(uv)\,du\,dv =$$

$$\underset{\uparrow}{=} \frac{1}{2}\int_1^2\left(\int_1^2 u\sin(uv)\,dv\right)du = \frac{1}{2}\int_1^2\Big[-\cos(uv)\Big]_{v=1}^2 du = \frac{1}{2}\int_1^2(-\cos 2u + \cos u)\,du = \frac{1}{2}\left[\sin u - \frac{\sin 2u}{2}\right]_1^2 =$$

lättast integrera v först (u inre derivata)

$$= \frac{1}{2}\left(\sin 2 - \frac{\sin 4}{2} - \sin 1 + \frac{\sin 2}{2}\right) = \frac{3\sin 2}{4} - \frac{\sin 4}{4} - \frac{\sin 1}{2}$$

6.29a $A(D) = \iint_D 1\,dxdy$, area av D (definition)



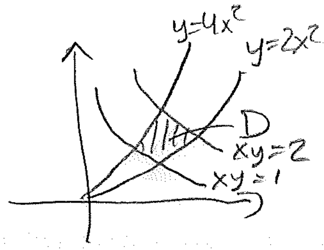
$D: |x+2y| + |3x-y| \le 1$. Sätt $\begin{cases} u = x+2y \\ v = 3x-y \end{cases}$ (linjärt byte) $\Rightarrow E: |u|+|v| \le 1$

$$\frac{d(x,y)}{d(u,v)} = \frac{1}{\frac{d(u,v)}{d(x,y)}} = \frac{1}{\begin{vmatrix} 1 & 2 \\ 3 & -1 \end{vmatrix}} = \frac{1}{-7}$$

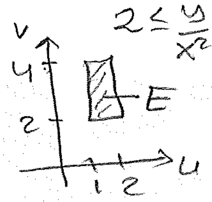
E kvadrat, sida $\sqrt{2}$

$$\Rightarrow A(D) = \iint_E \left|\frac{d(x,y)}{d(u,v)}\right| dudv = \frac{1}{7}\iint_E dudv = \frac{1}{7}A(E) = \frac{1}{7}(\sqrt{2})^2 = \underline{\frac{2}{7}}$$

6.29b $A(D) = \iint_D 1\,dxdy$ $\quad D: 1 \le xy \le 2$, $\underbrace{2x^2 \le y \le 4x^2}_{2 \le \frac{y}{x^2} \le 4}$



Med $\begin{cases} u = xy \\ v = y/x^2 \end{cases}$ fås ny mängd $\begin{cases} 1 \le u \le 2 \\ 2 \le v \le 4 \end{cases}$ (E)



$$\frac{d(u,v)}{d(x,y)} = \begin{vmatrix} y & x \\ -\frac{2y}{x^3} & \frac{1}{x^2} \end{vmatrix} = \frac{y}{x^2} + \frac{2y}{x^2} = \frac{3y}{x^2} = 3v > 0 \text{ i } E \Rightarrow \left|\frac{d(x,y)}{d(u,v)}\right| = \left|\frac{1}{3v}\right| = \frac{1}{3v} \Rightarrow$$

$$A(D) = \iint_E \cdot \frac{1}{3v}\,dudv = \frac{1}{3}\int_2^4 \left(\int_1^2 \frac{1}{v}\,du\right)dv = \frac{1}{3}\int_2^4 \left[\frac{1}{v}\cdot u\right]_{u=1}^{2} dv = \frac{1}{3}\int_2^4 \underbrace{\left(\frac{2}{v} - \frac{1}{v}\right)}_{1/v} dv =$$

$$= \frac{1}{3}\Big[\ln|v|\Big]_2^4 = \frac{1}{3}(\underbrace{\ln 4}_{\ln 2^2 = 2\ln 2} - \ln 2) = \underline{\frac{\ln 2}{3}}$$