2.65 Taylors formel till ordning 2 kring $(a,b)=(2,-1)$:

$$f(2+h,-1+k) = f(2,-1) + f'_x(2,-1)h + f'_y(2,-1)k + \frac{1}{2}\left(f''_{xx}(2,-1)h^2 + 2f''_{xy}(2,-1)hk + f''_{yy}(2,-1)k^2\right) +$$
$$+ \underbrace{\mathcal{O}\left(\sqrt{h^2+k^2}^{\,3}\right)}_{\text{resttermen}}$$

$$f(x,y) = \ln(2x^2+xy+y^2)$$

$$\Rightarrow f'_x = \frac{4x+y}{2x^2+xy+y^2}, \quad f'_y = \frac{x+2y}{2x^2+xy+y^2}$$, andraderivator med kvotregeln $\left(\frac{f}{g}\right)' = \frac{f'g-fg'}{g^2}$

$$\Rightarrow f''_{xx} = \frac{4\cdot(2x^2+xy+y^2)-(4x+y)(4x+y)}{(2x^2+xy+y^2)^2}, \quad f''_{xy} = (f'_x)'_y = \frac{1\cdot(2x^2+xy+y^2)-(4x+y)(x+2y)}{(2x^2+xy+y^2)}$$

$$f''_{yy} = \frac{2\cdot(2x^2+xy+y^2)-(x+2y)(x+2y)}{(2x^2+xy+y^2)^2}$$

I punkten $(2,-1)$:

$$f(2,-1) = \ln 7, \quad f'_x(2,-1) = \frac{7}{7} = 1, \quad f'_y(2,-1) = \frac{0}{7} = 0, \quad f''_{xx}(2,-1) = \frac{4\cdot 7 - 7\cdot 7}{7^2} = -\frac{3}{7}$$

$$f''_{xy}(2,-1) = \frac{1\cdot 7 - 7\cdot 0}{7^2} = \frac{1}{7}, \quad f''_{yy}(2,-1) = \frac{2\cdot 7 - 0\cdot 0}{7^2} = \frac{2}{7}$$

$\Rightarrow$

$$\underline{f(2+h,-1+k)} = \ln 7 + 1\cdot h + 0\cdot k + \frac{1}{2}\left(\frac{-3}{7}h^2 + 2\cdot\frac{1}{7}hk + \frac{2}{7}\cdot k^2\right) + \mathcal{O}\left(\sqrt{h^2+k^2}^{\,3}\right) =$$

$$\underline{= \ln 7 + h + \frac{1}{14}(-3h^2 + 2hk + 2k^2) + \mathcal{O}\left(\sqrt{h^2+k^2}^{\,3}\right)}$$

Man kan också använda $2+h=x,\ -1+k=y \Leftrightarrow h=x-2,\ k=y+1$ i svaret:

$$f(x,y) = \ln 7 + (x-2) + \frac{1}{14}\underbrace{\left(-3(x-2)^2 + 2(x-2)(y+1) + 2(y+1)^2\right)}_{\substack{\text{Multiplicera inte ihop parentesen och}\\ \text{utveckla inte kvadrater, man vill se}\\ \text{termer med } x-2 \text{ och } y+1}} + \mathcal{O}\left(\sqrt{(x-2)^2+(y+1)^2}^{\,3}\right)$$