

Exempel: Bestäm Maclaurinutvecklingen av ordning 3 till $f(x) = \ln(1 + \sin 3x)$

$$\begin{aligned}
 \underline{L:} \quad \ln(1 + \sin 3x) &= \left/ \begin{array}{l} \sin t = t - \frac{t^3}{6} + O(t^5) \\ \ln(1+s) = s - \frac{s^2}{2} + \frac{s^3}{3} + O(s^4) \end{array} \right/ = \\
 &= \ln\left(1 + 3x - \frac{(3x)^3}{6} + O((3x)^5)\right) = \ln\left(1 + \underbrace{3x - \frac{9x^3}{2} + O(x^5)}_s\right) \\
 &= \left(3x - \frac{9x^3}{2} + O(x^5)\right) - \frac{\left(3x - \frac{9x^3}{2} + O(x^5)\right)^2}{2} + \frac{\left(3x - \frac{9x^3}{2} + O(x^5)\right)^3}{3} \\
 &+ O\left(\left(3x - \frac{9x^3}{2} + O(x^5)\right)^4\right) = 3x - \frac{9x^3}{2} - \frac{9x^2}{2} + \frac{9x^3}{3} + O(x^4) \\
 &= \underline{\underline{3x - \frac{9x^2}{2} + \frac{9x^3}{2} + O(x^4)}}
 \end{aligned}$$

Motivering av ordokalkyl

$$O\left(\left(3x - \frac{9x^3}{2} + O(x^5)\right)^4\right) =$$

$$= b(x) \left(3x - \frac{9x^3}{2} + O(x^5)\right)^4 =$$

$$= b(x) \left(x \left(3 - \frac{9x^2}{2} + O(x^4)\right)\right)^4 =$$

$$= \underbrace{b(x) \left(3 - \frac{9x^2}{2} + O(x^4)\right)^4}_{b_1(x)} \cdot x^4 = O(x^4)$$

$b_1(x)$