Exempel B1.8. Avgör om

$$\int_1^\infty \frac{1+x}{\sqrt{x}+x^2}\,dx$$

är konvergent.

Lösning: Gen i ∞, positiv integrand.

$\left(\begin{array}{l} 1+x \approx x \\ \sqrt{x}+x^2 \approx x \end{array} \right.$ då $x \approx \infty$. $\frac{1+x}{\sqrt{x}+x^2} \approx \frac{x}{x^2} = \frac{1}{x}$ då $x \approx \infty$. $\left. \int_1^\infty \frac{1}{x}\,dx = \infty. \right)$

$1+x \geq x$, $1<x<\infty$

$\sqrt{x}+x^2 \leq x^2+x^2 = 2x^2$, $1<x<\infty$.

$\frac{1+x}{\sqrt{x}+x^2} \geq \frac{x}{2x^2} = \frac{1}{2x}$, $1<x<\infty$.

$$\int_1^\infty \frac{1+x}{\sqrt{x}+x^2}\,dx \geq \int_1^\infty \frac{1}{2x}\,dx = \lim_{t\to\infty} \int_1^t \frac{1}{2x}\,dx = \lim_{t\to\infty} \left[\frac{\ln|x|}{2}\right]_1^t = \infty.$$

SVAR: Divergent