Exempel: Lös $y + 4\int_x^3 y(t)\,dt = x^2 - 2.$

Lösning:

$$\frac{d}{dx}\left(y + 4\int_x^3 y(t)\,dt\right) = y' + 4\frac{d}{dx}\int_x^3 y(t)\,dt =$$

$$= \Big/ Y' = y,\ \frac{d}{dx}\int_x^3 y(t)\,dt = \frac{d}{dx}(Y(3) - Y(x)) = -Y'(x) = -y(x) \Big/$$

$$= y' - 4y = \frac{d}{dx}(x^2 - 2) = 2x.$$

$x = 3$ ger $\quad y(3) + 4\int_3^3 y(t)\,dt = y(3) = 3^2 - 2 = 7.$

$$\begin{cases} y' - 4y = 2x \\ y(3) = 7 \end{cases}.$$

$(-4x)' = -4$ ger I.F. e^{-4x}.

$$(e^{-4x}y)' = e^{-4x}y' - 4e^{-4x}y = e^{-4x}(y' - 4y) = e^{-4x}2x$$

$$\Leftrightarrow e^{-4x}y = \int e^{-4x} \cdot 2x\,dx = \ldots = -\frac{xe^{-4x}}{2} - \frac{e^{-4x}}{8} + C$$

$$y = -\frac{x}{2} - \frac{1}{8} + Ce^{4x}.$$

$y(3) = 7$: $7 = -\frac{3}{2} - \frac{1}{8} + Ce^{12}$; $C = \frac{69e^{-12}}{8}$

SVAR: $y = -\frac{x}{2} - \frac{1}{8} + \frac{69}{8} e^{4x-12}$