Exempel: Bestäm alla reella lösningar till

$$y^{(4)} - 2y^{(3)} + y'' + 2y' - 2y = -2x^3 + 6x^2 + 6x - 12.$$

Lösning: $y = y_h + y_p$

H.L. $p(r) = r^4 - 2r^3 + r^2 + 2r - 2 = 0$

$p(1) = 0.$ $\quad p(r) = (r-1)(r^3 - r^2 + 2) =$

$= (r-1)(r+1)(r^2 - 2r + 2)$

$r^2 - 2r + 2 = 0 \iff r = 1 \pm \sqrt{1-2} = 1 \pm i$

$r_1 = 1,\ r_2 = -1,\ r_3 = 1+i,\ r_4 = 1-i$

$$\left(y_h = C_1 e^{r_1 x} + C_2 e^{r_2 x} + C_3 e^{r_3 x} + C_4 e^{r_4 x} = \right.$$

$$\left. = C_1 e^{x} + C_2 e^{-x} + C_3 e^{(1+i)x} + C_4 e^{(1-i)x} \right)$$

$$\left(e^{(1+i)x} = e^{x}(\cos x + i \sin x),\ e^{(1-i)x} = e^{x}(\cos x - i \sin x) \right)$$

$$y_h = D_1 e^{x} + D_2 e^{-x} + D_3 e^{x} \cos x + D_4 e^{x} \sin x$$

P.L.

$$y_p = Ax^3 + Bx^2 + Cx + D$$

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$$y_p'' = 6Ax + 2B, \quad y_p''' = 6A, \quad y_p^{(4)} = 0.$$

$$y_p^{(4)} - 2y_p''' + y_p'' + 2y_p' - 2y_p =$$

$$= 0 - 2\cdot 6A + (6Ax + 2B) + 2(3Ax^2 + 2Bx + C) -$$

$$2(Ax^3 + Bx^2 + Cx + D) = -2Ax^3 + (6A - 2B)x^2 +$$

$$+ (6A + 4B - 2C)x + (-12A + 2B + 2C - 2D) = -2x^3 + 6x^2 + 6x - 12.$$

$$\begin{cases} -2A = -2 \\ 6A - 2B = 6 \\ 6A + 4B - 2C = 6 \\ -12A + 2B + 2C - 2D = -12 \end{cases} \iff \dots \iff \begin{cases} A = 1 \\ B = 0 \\ C = 0 \\ D = 0 \end{cases}.$$

$$\begin{cases} -2A = -2 \\ 6A - 2B = 6 \\ 6A + 4B - 2C = 6 \\ -12A + 2B + 2C - 2D = -12 \end{cases} \iff \ldots \iff \begin{cases} A = 1 \\ B = 0 \\ C = 0 \\ D = 0 \end{cases}.$$

SVAR!

$$y = D_1 e^{x} + D_2 e^{-x} + D_3 e^{x}\cos x + D_4 e^{x}\sin x + x^3. \qquad D_1, D_2, D_3, D_4 \in \mathbb{R}.$$