

Exempel: Lös $y'' - y = e^{2x} + \sin x$

Lösning: $r^2 - 1 = 0 \Leftrightarrow r = \pm 1$, så $y_h = Ae^x + Be^{-x}$.

$L(y) = y'' - y$. $y_p = y_1 + y_2$ där $L(y_1) = e^{2x}$, $L(y_2) = \sin x$
 $\Rightarrow L(y_p) = e^{2x} + \sin x$.

$$y_1 = ae^{2x}, \quad y_1' = 2ae^{2x}, \quad y_1'' = 4ae^{2x}.$$

$$y_1'' - y_1 = 4ae^{2x} - ae^{2x} = 3ae^{2x} = e^{2x} \Leftrightarrow 3a = 1 \Leftrightarrow a = 1/3.$$

$$y_2 = b\cos x + c\sin x, \quad y_2' = -b\sin x + c\cos x, \quad y_2'' = -b\cos x - c\sin x.$$

$$y_2'' - y_2 = (-b\cos x - c\sin x) - (b\cos x + c\sin x) = -2b\cos x - 2c\sin x = \sin x$$

$$\Leftrightarrow \begin{cases} -2b = 0 \\ -2c = 1 \end{cases}$$

$$\Leftrightarrow \begin{cases} b = 0 \\ c = -1/2 \end{cases}.$$

$$\text{SVAR: } y = y_h + y_1 + y_2 \\ = Ae^x + Be^{-x} + \frac{1}{3}e^{2x} - \frac{1}{2}\sin x.$$