

Exempel: Lös  $y'' - y = x e^x$ .

Lösning:  $r^2 - 1 = 0 \Leftrightarrow r = \pm 1$  ger  $y_h = A e^x + B e^{-x}$ .

$$y = z e^x \quad (z = z(x)). \quad y' = z' e^x + z e^x, \quad y'' = z'' e^x + 2z' e^x + z e^x.$$

$$y'' - y = ((z'' + 2z' + z) - z) e^x = (z'' + 2z') e^x = x e^x$$

$$\Leftrightarrow z'' + 2z' = x$$

$z_p$  ger  $y_p = z_p e^x$ .

$$z_p = ax^2 + bx, \quad z'_p = 2ax + b, \quad z''_p = 2a.$$

$$z''_p + 2z'_p = 2a + 2(2ax + b) = 4ax + (2a + 2b) = x$$

$$\Leftrightarrow \begin{cases} 4a = 1 \\ 2a + 2b = 0 \end{cases} \Leftrightarrow \begin{cases} a = 1/4 \\ b = -1/4. \end{cases} \quad \text{SVAR: } y = y_h + y_p =$$
$$= A e^x + B e^{-x} + \left( \frac{x^2}{4} - \frac{x}{4} \right) e^x.$$