Exempel: Lös $y'' + y = \sin x$

Lösning: $r^2 + 1 = 0 \Leftrightarrow r = \pm i$. $y_h = A\cos x + B\sin x$.

($y_p = a\cos x + b\sin x$ funkar ej!)

$\sin x = \mathrm{Im}(e^{ix})$

$w'' + w = e^{ix} \Rightarrow y_p = \mathrm{Im}(w_p)$.

$w = ze^{ix}$, $w' = z'e^{ix} + ize^{ix}$, $w'' = z''e^{ix} + 2iz'e^{ix} - ze^{ix}$.

$w'' + w = ((z'' + 2iz' - z) + z)e^{ix} = (z'' + 2iz')e^{ix} = e^{ix}$

$\Leftrightarrow z'' + 2iz' = 1 \Leftarrow z_p' = 1/2i \Leftarrow z_p = -\frac{ix}{2}$.

$w_p = -\frac{ix}{2}e^{ix} = -\frac{ix}{2}(\cos x + i\sin x) = \frac{x}{2}\sin x - \frac{ix}{2}\cos x$

$$w'' + w = ((z'' + 2iz' - z) + z)e^{ix} = (z'' + 2iz')e^{ix} = e^{ix}$$

$$\Leftrightarrow \quad z'' + 2iz' = 1 \;.\Leftarrow\; z_p' = 1/2i \Leftarrow z_p = \frac{-ix}{2}.$$

$$w_p = -\frac{ix}{2}e^{ix} = \frac{-ix}{2}(\cos x + i\sin x) = \frac{x}{2}\sin x - \frac{ix}{2}\cos x$$

$$y_p = \operatorname{Im}(w_p) = -\frac{x}{2}\cos x.$$

$$y = y_h + y_p = A\cos x + B\sin x - \frac{x}{2}\cos x.$$