Exempel

Bestäm den allmänna lösningen till

$$y^{(4)} - 2y^{(3)} + y'' + 2y' - 2y = 0.$$

Svaret ska ges på reell form.

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$$y = C_1 \mathrm{e}^{x} + C_2 \mathrm{e}^{-x} + C_3 \mathrm{e}^{x} \cos x + C_4 \mathrm{e}^{x} \sin x.$$

Svar: $y = C_1 \mathrm{e}^{x} + C_2 \mathrm{e}^{-x} + C_3 \mathrm{e}^{x} \cos x + C_4 \mathrm{e}^{x} \sin x.$

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$$D_3\mathrm{e}^{(1+i)x} + D_4\mathrm{e}^{(1-i)x}$$
$$= D_3\mathrm{e}^{x}(\cos x + i\sin x) + D_4\mathrm{e}^{x}(\cos(-x) + i\sin(-x))$$

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&D_3 e^{(1+i)x} + D_4 e^{(1-i)x} \\
&= D_3 e^{x}(\cos x + i \sin x) + D_4 e^{x}(\cos(-x) + i \sin(-x)) \\
&= D_3 e^{x}(\cos x + i \sin x) + D_4 e^{x}(\cos x - i \sin x)
\end{aligned}$$

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&= (D_3 + D_4) e^{x} \cos x + (iD_3 - iD_4) e^{x} \sin x
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&= D_3 e^{x}(\cos x + i \sin x) + D_4 e^{x}(\cos x - i \sin x) \\
&= (D_3 + D_4) e^{x} \cos x + (iD_3 - iD_4) e^{x} \sin x \\
&= C_3 e^{x} \cos x + C_4 e^{x} \sin x.
\end{aligned}$$