13.1)

a) Volym av ett halvklot är $\frac{2\pi \cdot R^3}{3}$

Så massa blir $\frac{2\pi R^3 \rho}{3}$

b) $\rho(x,y,z) = k\sqrt{x^2+y^2+z^2}$.

$$\text{Massa} = \iiint_D \rho(x,y,z) = k\int_0^{\pi/2}\left(\int_0^{2\pi}\left(\int_0^R r\cdot r^2\sin\theta\, dr\right)d\phi\right)d\theta$$

$$= k\int_0^{\pi/2}\sin\theta\, d\theta \cdot 2\pi \cdot \int_0^R r^3\, dr = \frac{2\pi k R^4}{4} = \underline{\underline{\frac{\pi k R^4}{2}}}$$

13.2) Silon ges i koordinaterna

$$\begin{cases} x = \rho\cos\phi \\ y = \rho\sin\phi \\ z = z \end{cases}$$

av $0 \le z \le 8,\ 0 \le \rho \le 3,\ 0 \le \phi \le 2\pi$

$dx\,dy\,dz = \rho\, d\rho\, d\phi\, dz$.

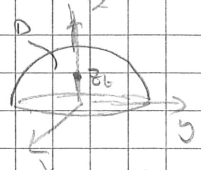
$$\int_0^8\left(\int_0^{2\pi}\left(\int_0^3 (10-z)^2(4-\rho)\rho\, d\rho\right)d\phi\right)dz = 2\pi\int_0^8 (10-z)^2\, dz \cdot \int_0^3 (4-\rho)\rho\, d\rho =$$

$$= 2\pi\left[-\frac{(10-z)^3}{3}\right]_0^8 \cdot \left[2\rho^2 - \frac{\rho^3}{3}\right]_0^3 = 18\pi\left(\frac{1000}{3} - \frac{8}{3}\right) = \underline{\underline{5952\pi\,\text{kg}}}.$$

6.41)

a)

13.3) a)



$$z_t = \frac{\iiint_D z\,dxdydz}{\iiint_D dxdydz} = \frac{\int_0^{\pi/2}\left(\int_0^{2\pi}\left(\int_0^R r\cos\theta \cdot r^2\sin\theta\,dr\right)d\varphi\right)d\theta}{\frac{2\pi R^3}{3}} =$$

$$= \frac{2\pi\int_0^{\pi/2}\cos\theta\sin\theta\,d\theta \int_0^R r^3\,dr}{\frac{2\pi R^3}{3}} = \frac{3}{R^3}\cdot\int_0^{\pi/2}\frac{\sin 2\theta}{2}\,d\theta\cdot\frac{R^4}{4} =$$

$$= \frac{3R}{4}\left[-\frac{\cos 2\theta}{4}\right]_0^{\pi/2} = \frac{3R}{8}$$

SVAR: $(0, 0, \frac{3R}{8})$.

b)

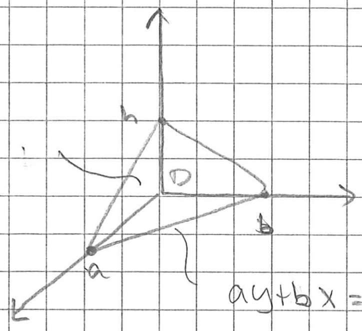
$$z_t = \frac{\iiint_D z\,k\sqrt{x^2+y^2+z^2}\,dxdydz}{\iiint_D k\sqrt{x^2+y^2+z^2}\,dxdydz} =$$

$$= \frac{k\int_0^{\pi/2}\left(\int_0^{2\pi}\left(\int_0^R r\cos\theta\, r\cdot r^2\sin\theta\,dr\right)d\varphi\right)d\theta}{k\int_0^{\pi/2}\left(\int_0^{2\pi}\left(\int_0^R r\cdot r^2\sin\theta\,dr\right)d\varphi\right)d\theta} =$$

$$= \frac{\int_0^{\pi/2}\cos\theta\sin\theta\,d\theta\cdot\int_0^R r^4\,dr}{\int_0^{\pi/2}\sin\theta\,d\theta\cdot\int_0^R r^3\,dr} = \frac{4R^5}{5R^4}\cdot\frac{\frac{1}{2}}{1} = \frac{2R^5}{5R^4} = \frac{2R}{5}$$

SVAR: $(0, 0, \frac{2R}{5})$.

(13.4)



$D = \{(x,y,z) : 0 \le x \le a,\ 0 \le y \le b - \frac{bx}{a},\ 0 \le z \le h - \frac{h}{a}x - \frac{h}{b}y\}$

$ay + bx = ab \Leftrightarrow y = b - \frac{bx}{a}$

$z = \alpha x + \beta y + \gamma$ som går genom $(a,0,0)$, $(0,b,0)$ och $(0,0,h)$

ger $h = \gamma$, $0 = \alpha a + \gamma$, $0 = \beta b + \gamma$,

d.v.s. $\alpha = -\frac{h}{a}$, $\beta = -\frac{h}{b}$.

$$\iiint_D z\,dx\,dy\,dz = \int_0^a \left(\int_0^{b-\frac{bx}{a}} \left(\int_0^{h-\frac{hx}{a}-\frac{hy}{b}} z\,dz \right) dy \right) dx =$$

$$= \frac{1}{2}\int_0^a \left(\int_0^{b-\frac{bx}{a}} \left(h - \frac{hx}{a} - \frac{hy}{b}\right)^2 dy \right) dx$$

$$= \frac{1}{2}\int_0^a \left[-\frac{b\left(h - \frac{hx}{a} - \frac{hy}{b}\right)^3}{h \cdot 3} \right]_{y=0}^{b-\frac{bx}{a}} dx =$$

$$= \frac{1}{2}\int_0^a \frac{b}{3h}\left(h - \frac{hx}{a}\right)^3 dx = \frac{1}{2}\cdot\frac{b}{3h}\left[-\frac{a}{4h}\left(h - \frac{hx}{a}\right)^4\right]_0^a$$

$$= \frac{abh^2}{24}$$

$$\iiint_D dx\,dy\,dz = \int_0^a \left(\int_0^{b-\frac{bx}{a}} \left(\int_0^{h-\frac{hx}{a}-\frac{hy}{b}} dz \right) dy \right) dx =$$

$$= \int_0^a \left(\int_0^{b-\frac{bx}{a}} \left(h - \frac{hx}{a} - \frac{hy}{b}\right) dy \right) dx = \int_0^a \left[\frac{-b}{2h}\left(h - \frac{hx}{a} - \frac{hy}{b}\right)^2 \right]_{y=0}^{b-\frac{bx}{a}} dx$$

$$= \int_0^a \frac{b}{2h}\left(h - \frac{hx}{a}\right)^2 dx = \left[\frac{-ab}{6h^2}\left(h - \frac{hx}{a}\right)^3 \right]_0^a = \frac{abh}{6}$$

$$z_t = \frac{abh^2}{24} \Big/ \frac{abh}{6} = \underline{\underline{\frac{h}{4}}}.$$