7.1) $\begin{cases} x = 3t\cos\psi \\ y = 2t\sin\psi \end{cases}$

$$\frac{\partial(x,y)}{\partial(t,\psi)} = \begin{pmatrix} \frac{\partial x}{\partial t} & \frac{\partial x}{\partial \psi} \\ \frac{\partial y}{\partial t} & \frac{\partial y}{\partial \psi} \end{pmatrix} = \begin{pmatrix} 3\cos\psi & -3t\sin\psi \\ 2\sin\psi & 2t\cos\psi \end{pmatrix}$$

$$\frac{d(x,y)}{d(t,\psi)} = \begin{vmatrix} 3\cos\psi & -3t\sin\psi \\ 2\sin\psi & 2t\cos\psi \end{vmatrix} =$$

$$= 6t\cos^2\psi + 6t\sin^2\psi = \underline{\underline{6t}}$$

7.2) $\begin{cases} u = x + y + 2z \\ v = 3x - 2y + z \\ w = 5z - x - y \end{cases}$

$$\frac{\partial(u,v,w)}{\partial(x,y,z)} = \begin{pmatrix} \frac{\partial u}{\partial x} & \frac{\partial u}{\partial y} & \frac{\partial u}{\partial z} \\ \frac{\partial v}{\partial x} & \frac{\partial v}{\partial y} & \frac{\partial v}{\partial z} \\ \frac{\partial w}{\partial x} & \frac{\partial w}{\partial y} & \frac{\partial w}{\partial z} \end{pmatrix} = \begin{pmatrix} 1 & 1 & 2 \\ 3 & -2 & 1 \\ -1 & -1 & 5 \end{pmatrix}$$

$$\frac{d(u,v,w)}{d(x,y,z)} = \begin{vmatrix} 1 & 1 & 2 \\ 3 & -2 & 1 \\ -1 & -1 & 5 \end{vmatrix} \begin{matrix} -3 & 1 \end{matrix} = \begin{vmatrix} 1 & 1 & 2 \\ 0 & -5 & -5 \\ 0 & 0 & 7 \end{vmatrix} = -35 \neq 0.$$

Ja, inverterbar.

7.3) $f(x,y) = x^3 + y^3 + xy - x - y = 0$

$$\frac{\partial f}{\partial y} = 3y^2 + x - 1$$

a) $\frac{\partial f}{\partial y}(0,0) = -1 \neq 0$

b) $\frac{\partial f}{\partial y}(0,1) = 3 - 1 = 2 \neq 0$

c) $\frac{\partial f}{\partial y}(0,-1) = 3 - 1 = 2 \neq 0$

$$\frac{d}{dx}\left(x^3 + y^3(x) + xy(x) - x - y(x)\right) = 3x^2 + 3y^2(x)y'(x) + y(x) + xy'(x)$$
$$- 1 - y'(x) = 0$$

$\Leftrightarrow \quad (3y^2(x) + x - 1)y'(x) + 3x^2 + y(x) - 1 = 0$

$\Leftrightarrow \quad y'(x) = \dfrac{1 - 3x^2 - y}{3y^2 + x - 1}$

a) $y(0) = 0$, $y'(0) = -1$, b) $y(0) = 1$, $y'(0) = 0$, c) $y(0) = -1$, $y'(0) = 1$.

7.4) $f(x,y) = x^3 - 3xy^2 - 1 = 0$,

a) $f(1,0) = 1 - 1 = 0$ o.k.

$\frac{\partial f}{\partial x} = 3x^2 - 3y^2$, $\frac{\partial f}{\partial x}(1,0) = 3 \neq$ o.k

$$\frac{d}{dy} f(x(y),y) = \frac{\partial f}{\partial x}\frac{dx}{dy} + \frac{\partial f}{\partial y}\frac{dy}{dy} = (3x^2 - 3y^2)x'(y) + (-6xy) = 0$$

$x'(y) = \dfrac{6xy}{3x^2 - 3y^2}$ $\quad x(0) = 1$, $x'(0) = 0$.

b) Eftersom f är C^2 blir x C^2.

$$x''(y) = \frac{d}{dy}\frac{6x(y)y}{3x^2(y) - 3y^2} = \frac{(6x'(y)y + 6x)(3x^2 - 3y^2) - 6xy(6xx'(y) - 6y)}{(3x^2 - 3y^2)^2}$$

$x''(0) = \dfrac{6 \cdot 3}{3^2} = \underline{\underline{2}}$. Lok. Min.

c)

$3x^2 - 3y^2 = 0 \Leftrightarrow x^2 = y^2$

$$\begin{cases} x^2 = y^2 \\ f(x,y) = 0 \end{cases} \Leftrightarrow \begin{cases} y = \pm x \\ x^3 - 3x^3 - 1 = 0 \end{cases} \Leftrightarrow \begin{cases} y = \pm x \\ 2x^3 = -1 \end{cases}$$

Två punkter $\left(\dfrac{-1}{\sqrt[3]{2}}, \pm\dfrac{1}{\sqrt[3]{2}}\right)$

$\frac{\partial f}{\partial y} = -6xy \neq 0$ i bägge dessa, så vi kan lösa ut $y(x)$ lokalt.

7.5)

$$f(x,y,z) = xy - (x+y)z^2 + 1 - \tan 2z$$

$$f(1,-1,\pi) = -1 + 1 + \tan 2\pi = 0 \quad \text{ok.}$$

$$\frac{\partial f}{\partial z} = -2(x+y)z - 2(1+\tan^2 2z)$$

$$\frac{\partial f}{\partial z}(1,-1,\pi) = -2 \neq 0 \quad \text{o.k.}$$

$$z(1,-1) = \pi$$

$$\frac{\partial}{\partial x}\left(xy - (x+y)z^2(x,y) + 1 - \tan(2z(x,y))\right) =$$

$$= y - z^2 - 2(x+y)z\,z'_x(x,y) - 2z'_x(x,y)(1+\tan^2 2z)) = 0$$

$$\Leftrightarrow \quad y - z^2 = z'_x\left(2(x+y)z + 2(1+\tan^2 2z)\right)$$

$$\Leftrightarrow \quad z'_x = \frac{y-z^2}{2(x+y)z+2(1+\tan^2 2z)}$$

$$z'_x(1,-1) = \frac{-1-\pi^2}{2}$$

$$\frac{\partial}{\partial y}\left(xy - (x+y)z^2(x,y) + 1 - \tan(2z(x,y))\right) =$$

$$= x - z^2 - 2(x+y)z\,z'_y(x,y) - 2z'_y(x,y)(1+\tan^2 2z)) = 0$$

$$\Leftrightarrow \quad z'_y = \frac{x-z^2}{2(x+y)z+2(1+\tan 2z)} \qquad z'_y(1,-1) = \frac{1-\pi^2}{2}$$

7.6) $\begin{cases} f_1(x,y,z) = x+y+z-6 = 0 \\ f_2(x,y,z) = xyz-6 = 0 \end{cases}$

$$\begin{vmatrix} \frac{\partial f_1}{\partial x} & \frac{\partial f_1}{\partial z} \\ \frac{\partial f_2}{\partial x} & \frac{\partial f_2}{\partial z} \end{vmatrix} = \begin{vmatrix} 1 & 1 \\ yz & xy \end{vmatrix} = xy - yz \quad i\ (1,2,3) = 2-6 = -4 \neq 0$$

$\begin{cases} f_1(1,2,3) = 0 \\ f_2(1,2,3) = 0 \end{cases}$ ok.

$x(2) = 1, \quad z(2) = 3$

$\begin{cases} x(y) + y + z(y) = 6 \\ x(y)\,y\,z(y) = 6 \end{cases} \Rightarrow \begin{cases} \frac{d}{dy}(x(y)+y+z(y)) = x'(y)+1+z'(y) = 0 \\ \frac{d}{dy}(x(y)\,y\,z(y)) = x'(y)yz + xz + xyz'(y) = 0 \end{cases}$

$\Leftrightarrow \begin{cases} x'(y) + z'(y) = -1 \\ yz\,x'(y) + xy\,z'(y) = -xz \end{cases}$ $-yz$

$\Leftrightarrow \begin{cases} x'(y) + z'(y) = -1 \\ (xy-yz)z'(y) = yz - xz \end{cases} \Leftrightarrow \begin{cases} x'(y) = -1 - \frac{yz-xz}{xy-yz} \\ z'(y) = \frac{yz-xz}{xy-yz} \end{cases}$

$\begin{cases} x'(2) = -1 - \frac{6-3}{-4} = -\frac{1}{4} \\ z'(2) = \frac{6-3}{-4} = -\frac{3}{4}. \end{cases}$

7.7) $\begin{cases} f_1(x,y,z) = x^2+y^2-z^2-2 = 0 \\ f_2(x,y,z) = x+y-2e^z = 0 \end{cases}$

$f_1(1,1,0) = f_2(1,1,0) = 0$ o.k.

$$\begin{vmatrix} \frac{\partial f_1}{\partial y} & \frac{\partial f_1}{\partial z} \\ \frac{\partial f_2}{\partial y} & \frac{\partial f_2}{\partial z} \end{vmatrix} = \begin{vmatrix} 2y & -2z \\ 1 & -2e^z \end{vmatrix} =$$

$= -4ye^z + 2z \cdot 1$ $(1,1,0)$ får vi $-4 \neq 0.$

$y(x), z(x)$ lokalt kring $(1,1,0)$.

$\begin{cases} 2x + 2yy'(x) - 2zz'(x) = 0 \\ 1 + y'(x) - 2z'(x)e^z = 0 \end{cases}$

i $(1,1,0)$

$\begin{cases} 2 + 2y'(1) = 0 \\ 1 + y'(1) - 2z'(1) = 0 \end{cases} \Rightarrow \begin{array}{l} y'(1) = -1 \\ z'(1) = 0 \end{array}$

Tangentvektor $(1,-1,0)$.

7.8)

$t = v - e\sin v$, $0 \le e < 1$ konstant.

$f(t,v) = t - v + e\sin v = 0$

$\frac{\partial f}{\partial v} = -1 + e\cos v < 0$ så implicita funktionssatsen

ger att $v(t)$ existerar lokalt som C^1 funktion.

Eftersom $\frac{\partial f}{\partial v} < 0$ $f(t,v) \to -\infty$ då $v \to \infty$, $f(t,v) \to +\infty$ då $v \to -\infty$

finns unik lösning $v(t)$ för varje t.

$1 - v'(t) + e\,v'(t)\cos v = 0$

$\Rightarrow v'(t) = \frac{1}{1 - e\cos v} > 0$.

7.9)

$(y^2 + z^4)x + x^5 = f(x,y,z) = 1$

$\frac{\partial f}{\partial x} = (y^2+z^4) + 5x^4 \geq 0$, > 0 då $(x,y,z) \neq (0,0,0)$ så

spec. på $f = 1$.

$f(x,y,z) \to -\infty$ då $x \to -\infty$, $f(x,y,z) \to +\infty$ då $x \to +\infty$

så finns unik lösning för varje (y,z).

$\frac{\partial}{\partial y}\left((y^2+z^4)x(y,z) + x^5(y,z)\right) =$

$= 2yx + (y^2+z^4)x'_y + 5x^4 x'_y = 0$

$x'_y = \frac{-2xy}{5x^4+y^2+z^4}$

$\frac{\partial}{\partial z}\left((y^2+z^4)x(y,z) + x^5(y,z)\right) =$

$= 4z^3 x + (y^2+z^4)x'_z + 5x^4 x'_z = 0$

$x'_z = \frac{-4z^3 x}{5x^4+y^2+z^4}$