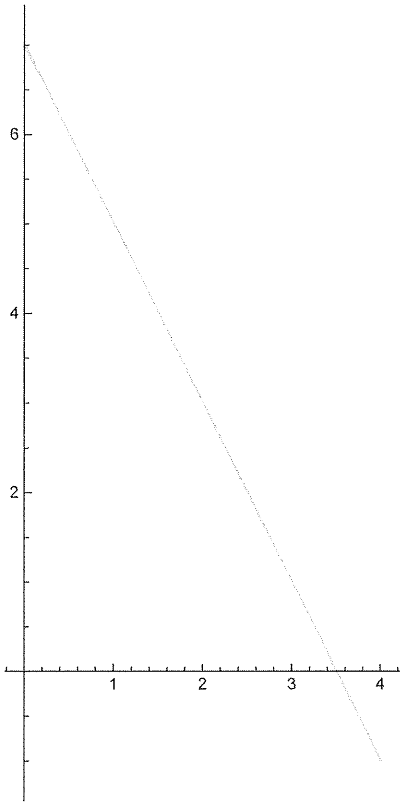
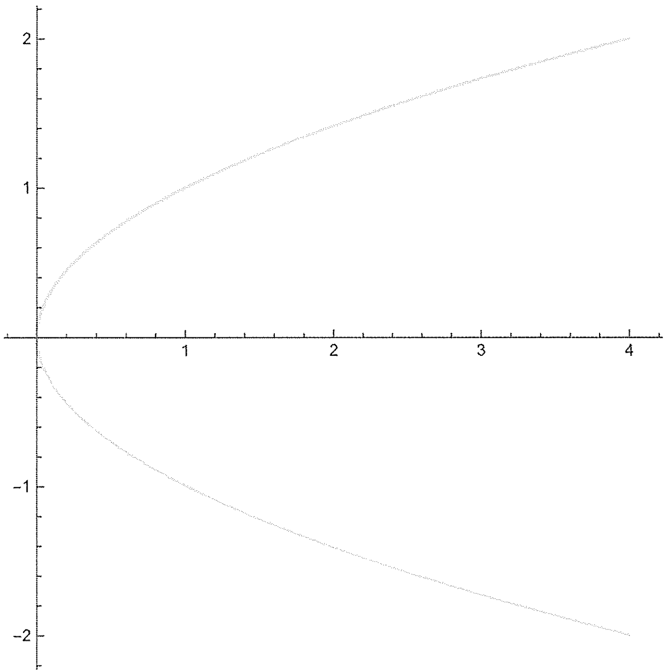
3.1 a : Rät linje y = ~~5~~ 7 – 2 x

Out[2]=



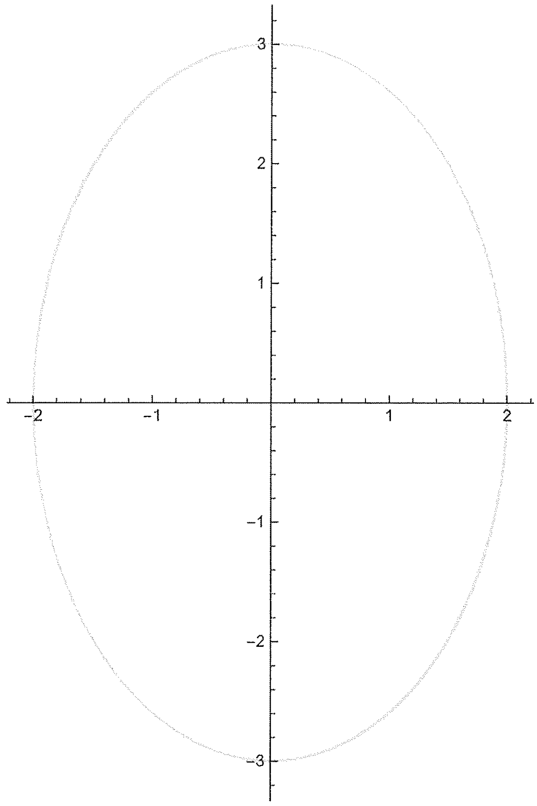
3.1 c : Kurvan x = y^2

Out[3]=



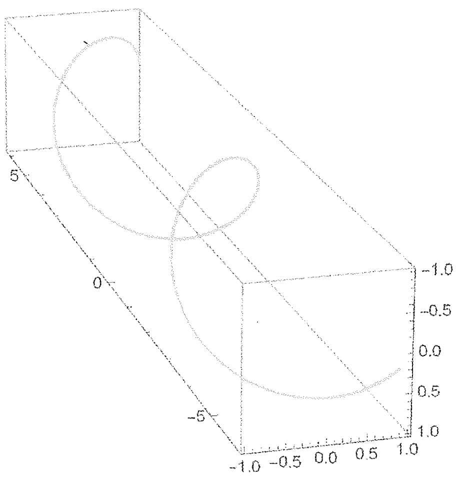
3.2 a : (3 x) ^2 + (2 y) ^2 = (6 cost) ^2 + (6 sint) ^2 = 36 ger ellips

Out[4]=



3.2 c : Spiral runt z - axeln.

Out[7]=



3.4) $\bar{F}(s,t) = \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} se^t - 1 \\ \sin st \\ 2s + \arcsin t \end{pmatrix}$, $\bar{F}(1,0) = \begin{pmatrix} 0 \\ 0 \\ 2 \end{pmatrix}$

$$\frac{\partial \bar{F}}{\partial(s,t)} = \begin{pmatrix} \frac{\partial x}{\partial s} & \frac{\partial x}{\partial t} \\ \frac{\partial y}{\partial s} & \frac{\partial y}{\partial t} \\ \frac{\partial z}{\partial s} & \frac{\partial z}{\partial t} \end{pmatrix} = \begin{pmatrix} e^t & se^t \\ t\cos st & s\cos st \\ 2 & \frac{1}{\sqrt{1-t^2}} \end{pmatrix}$$

$$\frac{\partial \bar{F}}{\partial(s,t)}(1,0) = \begin{pmatrix} 1 & 1 \\ 0 & 1 \\ 2 & 1 \end{pmatrix}$$

$$\bar{F}(1+h, 0+k) = \bar{F}(1,0) + \frac{\partial \bar{F}}{\partial(s,t)}(1,0)\begin{pmatrix} h \\ k \end{pmatrix} + O(|(h,k)|^2)$$

$$= \begin{pmatrix} 0 \\ 0 \\ 2 \end{pmatrix} + h\begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix} + k\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} + O(|(h,k)|^2)$$

Tangentplan ges på parameterform av

$$\underline{\underline{(x,y,z) = (0,0,2) + h(1,0,2) + k(1,1,1)}}$$

Normal: $-(1,0,2) \times (1,1,1) = (2,-1,-1)$

$$(2,-1,-1)\cdot(x,y,z) = (2,-1,-1)\cdot(0,0,2) \Leftrightarrow \underline{\underline{2x - y - z = -2}}$$

3.6) $\begin{cases} x = 3t\cos\psi \\ y = 2t\sin\psi \end{cases}$

$$\frac{\partial(x,y)}{\partial(t,\psi)} = \begin{pmatrix} \frac{\partial x}{\partial t} & \frac{\partial x}{\partial \psi} \\ \frac{\partial y}{\partial t} & \frac{\partial y}{\partial \psi} \end{pmatrix} = \begin{pmatrix} 3\cos\psi & -3t\sin\psi \\ 2\sin\psi & 2t\cos\psi \end{pmatrix}$$

$$\frac{d(x,y)}{d(t,\psi)} = \begin{vmatrix} 3\cos\psi & -3t\sin\psi \\ 2\sin\psi & 2t\cos\psi \end{vmatrix} = 6t\cos^2\psi + 6t\sin^2\psi = \underline{\underline{6t}}$$

3.7) $\begin{cases} u = x+y+2z \\ v = 3x-2y+z \\ w = 5z-x-y \end{cases}$

$$\frac{\partial(u,v,w)}{\partial(x,y,z)} = \begin{pmatrix} \frac{\partial u}{\partial x} & \frac{\partial u}{\partial y} & \frac{\partial u}{\partial z} \\ \frac{\partial v}{\partial x} & \frac{\partial v}{\partial y} & \frac{\partial v}{\partial z} \\ \frac{\partial w}{\partial x} & \frac{\partial w}{\partial y} & \frac{\partial w}{\partial z} \end{pmatrix} = \begin{pmatrix} 1 & 1 & 2 \\ 3 & -2 & 1 \\ -1 & -1 & 5 \end{pmatrix}$$

$$\frac{d(u,v,w)}{d(x,y,z)} = \begin{vmatrix} 1 & 1 & 2 \\ 3 & -2 & 1 \\ -1 & -1 & 5 \end{vmatrix} \; -3 \; 1 \; = \begin{vmatrix} 1 & 1 & 2 \\ 0 & -5 & -5 \\ 0 & 0 & 7 \end{vmatrix} = -35 \neq 0.$$

Ja, inverterbar.

3.8) $\begin{cases} u = e^x + y \\ v = e^{-x} y \end{cases}$

a) $y = ve^x = u - e^x \Leftrightarrow (v+1)e^x = u \Rightarrow x = \ln\left(\frac{u}{1+v}\right)$

$y = v \cdot \frac{u}{1+v} = \frac{uv}{1+v}$

$\begin{cases} x = \ln\left(\frac{u}{1+v}\right) = \ln u - \ln(1+v) \\ y = \frac{uv}{1+v} \end{cases}$

b) $\frac{\partial(u,v)}{\partial(x,y)} = \begin{pmatrix} e^x & 1 \\ -e^{-x}y & e^{-x} \end{pmatrix}$ $\quad \frac{d(u,v)}{d(x,y)} = 1 + e^{-x}y = 1+v$

$\frac{\partial(x,y)}{\partial(u,v)} = \begin{pmatrix} 1/u & -\frac{1}{1+v} \\ \frac{v}{1+v} & \frac{u(1+v)-uv}{(1+v)^2} \end{pmatrix}$ $\quad \frac{d(x,y)}{d(u,v)} = \frac{1}{(1+v)^2} + \frac{v}{(1+v)^2} = \frac{1}{1+v}$

$\frac{d(u,v)}{d(x,y)} \cdot \frac{d(x,y)}{d(u,v)} = 1.$

3.9) $\begin{cases} u = e^x + y \\ v = 2x + e^y \end{cases}$

a) $$\frac{\partial(u,v)}{\partial(x,y)} = \begin{pmatrix} e^x & 1 \\ 2 & e^y \end{pmatrix}$$

$$\frac{\partial(u,v)}{\partial(x,y)}(1,0) = \begin{pmatrix} e & 1 \\ 2 & 1 \end{pmatrix} \qquad \frac{d(u,v)}{d(x,y)}(1,0) = e-2 \neq 0$$

$(u(1,0), v(1,0)) = (e,3)$.

Inversa funktionssatsen ger nu att det finns C^1-invers i någon omgivning till $(1,0)$.

b) $$\frac{\partial(x,y)}{\partial(u,v)} = \left(\frac{\partial(u,v)}{\partial(x,y)}\right)^{-1} = \begin{pmatrix} e^x & 1 \\ 2 & e^y \end{pmatrix}^{-1} = \frac{1}{e^x e^y - 2}\begin{pmatrix} e^y & -1 \\ -2 & e^x \end{pmatrix}$$

$$= \begin{pmatrix} x'_u & x'_v \\ y'_u & y'_v \end{pmatrix}$$

c) Eftersom $(x,y) \mapsto (u,v)$ är C^2 blir även inversen det.

$$z'_u = z'_x x'_u + z'_y y'_u$$

$$z'_v = z'_x x'_v + z'_y y'_v$$

$$z''_{uv} = (z'_u)'_v = (z'_x x'_u + z'_y y'_u)'_v =$$

$$= (z'_x x'_u + z'_y y'_u)'_x x'_v + (z'_x x'_u + z'_y y'_u)'_y y'_v$$

$$= /\text{ Med } z = x\ / = \left(\frac{e^y}{e^x e^y - 2}\right)'_x \cdot \left(\frac{-1}{e^x e^y - 2}\right) + \left(\frac{e^y}{e^x \cdot e^y - 2}\right)'_y \cdot \frac{e^x}{e^x e^y - 2}$$

$$= \frac{e^x e^{2y}}{(e^x e^y - 2)^3} + \frac{(e^y(e^x \cdot e^y - 2) - e^y \cdot e^x e^y)e^x}{(e^x e^y - 2)^3} = \frac{e^x e^{2y} - 2e^x e^y}{(e^x e^y - 2)^3}$$

$$= \frac{e^x e^y (e^y - 2)}{(e^x e^y - 2)^3} = x''_{uv}.$$

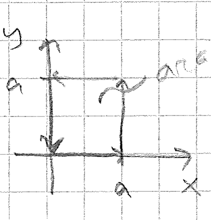
$$x''_{uv}(e,3) = \frac{e(1-2)}{(e-2)^3} = -\underline{\underline{\frac{e}{(e-2)^3}}}.$$

3.10) $\begin{cases} u = x^2 + 2y \\ v = x + y \end{cases}$

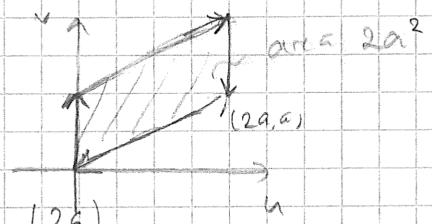
a) $\dfrac{\partial(u,v)}{\partial(x,y)} = \begin{pmatrix} 2x & 2 \\ 1 & 1 \end{pmatrix}$ $\quad \dfrac{d(u,v)}{d(x,y)}(0,0) = \begin{vmatrix} 0 & 2 \\ 1 & 1 \end{vmatrix} = -2 \neq 0$ ok.

b) $\begin{pmatrix} u(0+h, 0+k) \\ v(0+h, 0+k) \end{pmatrix} \approx \begin{pmatrix} u(0,0) \\ v(0,0) \end{pmatrix} + \dfrac{\partial(u,v)}{\partial(x,y)}(0,0)\begin{pmatrix} h \\ k \end{pmatrix} =$

$= \begin{pmatrix} 0 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 & 2 \\ 1 & 1 \end{pmatrix}\begin{pmatrix} h \\ k \end{pmatrix} = h\begin{pmatrix} 0 \\ 1 \end{pmatrix} + k\begin{pmatrix} 2 \\ 1 \end{pmatrix}$



$\begin{pmatrix} u(0,a) \\ v(0,a) \end{pmatrix} \approx a\begin{pmatrix} 2 \\ 1 \end{pmatrix}$



$\begin{pmatrix} u(a,a) \\ v(a,a) \end{pmatrix} \approx a\begin{pmatrix} 0 \\ 1 \end{pmatrix} + a\begin{pmatrix} 2 \\ 1 \end{pmatrix} = \begin{pmatrix} 2a \\ 2a \end{pmatrix}$

$\begin{pmatrix} u(a,0) \\ v(a,0) \end{pmatrix} \approx a\begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ a \end{pmatrix}$

c) Tecknet ger omvänd orientering.

$|-2| = 2$ förhållandet mellan areorna.

3.11) $(x,y) \mapsto (u,v)$ $\begin{cases} u = x^2 - y^2 \\ v = 2xy \end{cases}$ $x^2+y^2>0$

a) $\dfrac{\partial(u,v)}{\partial(x,y)} = \begin{pmatrix} 2x & -2y \\ 2y & 2x \end{pmatrix}$ $\dfrac{d(u,v)}{d(x,y)} = 4x^2+4y^2 > 0$ on $(x,y) \neq (0,0)$.

b) $(u(1,1), v(1,1)) = (0,2) = (u(-1,-1), v(-1,-1))$.

c) $x>0$: $\begin{cases} u = x^2-y^2 \\ v = 2xy \end{cases} \Leftrightarrow \begin{cases} u = x^2 - (v/2x)^2 \\ y = v/2x \end{cases} \Leftrightarrow$

$\Leftrightarrow \begin{cases} u = x^2 - \dfrac{v^2}{4x^2} \\ y = v/2x \end{cases} \Leftrightarrow \begin{cases} x^4 - x^2 u - \dfrac{v^2}{4} = 0 \\ y = v/2x \end{cases}$

$\Leftrightarrow \begin{cases} x^2 = \dfrac{u}{2} \stackrel{+}{(-)} \sqrt{\dfrac{u^2}{4} + \dfrac{v^2}{4}} \\ y = v/2x \end{cases} \Leftrightarrow \begin{cases} x = \sqrt{\dfrac{u}{2} + \sqrt{\dfrac{u^2}{4} + \dfrac{v^2}{4}}} \\ y = \dfrac{v}{2\sqrt{\dfrac{u}{2} + \sqrt{\dfrac{u^2}{4}+\dfrac{v^2}{4}}}} \end{cases}$

$$\frac{d(x,y)}{d(u,v)} = \frac{1}{4x^2+4y^2} = \frac{1}{4\left(\frac{u+\sqrt{u^2+v^2}}{2}\right) + 4\left(\frac{v}{2\sqrt{\frac{u+\sqrt{u^2+v^2}}{2}}}\right)^2} =$$

$$= \ldots = \frac{1}{4\sqrt{u^2+v^2}}.$$