Exempel

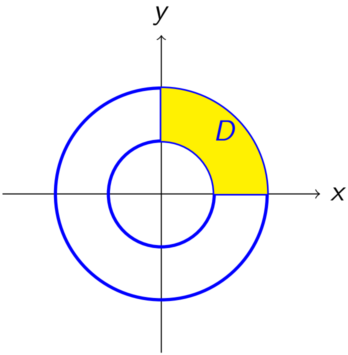
Beräkna

$$\iint_D \frac{xdxdy}{x^2+y^2}$$

där $D = \{(x, y) : 1 \leq x^2 + y^2 \leq 4, x \geq 0, y \geq 0\}$.

Lösning

$$D = \{(x, y) : 1 \leq x^2 + y^2 \leq 4, x \geq 0, y \geq 0\}.$$



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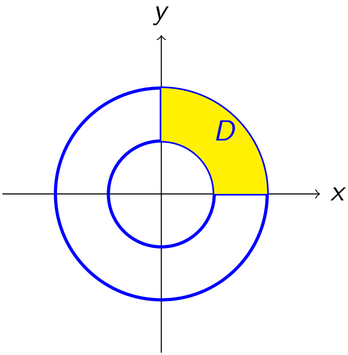

I polära koordinater $x = \rho \cos \varphi$, $y = \rho \sin \varphi$ ges D av att $1 \leq \rho \leq 2$ och $0 \leq \varphi \leq \pi/2$.

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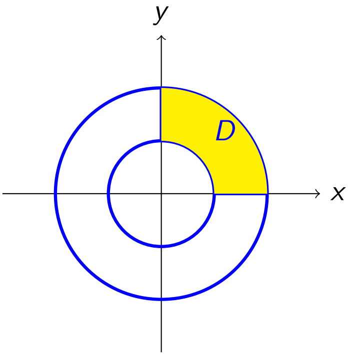

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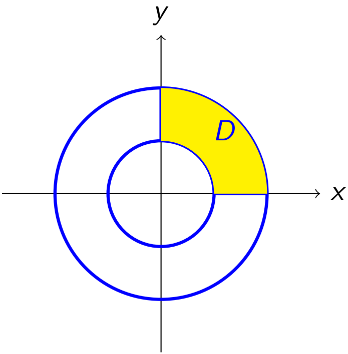

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$$\iint_D \frac{x}{x^2+y^2}dxdy = \int_0^{\pi/2}\left(\int_1^2 \frac{\rho\cos\varphi}{\rho^2}\rho d\rho\right)d\varphi$$

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$$\int_0^{\pi/2}\cos\varphi d\varphi$$

Lösning

$$\iint_D \frac{x}{x^2+y^2}\,dxdy = \int_0^{\pi/2} \left(\int_1^2 \frac{\rho\cos\varphi}{\rho^2} \rho d\rho \right) d\varphi =$$
$$\int_0^{\pi/2} \cos\varphi d\varphi = [\sin\varphi]_0^{\pi/2} = 1.$$