Exempel

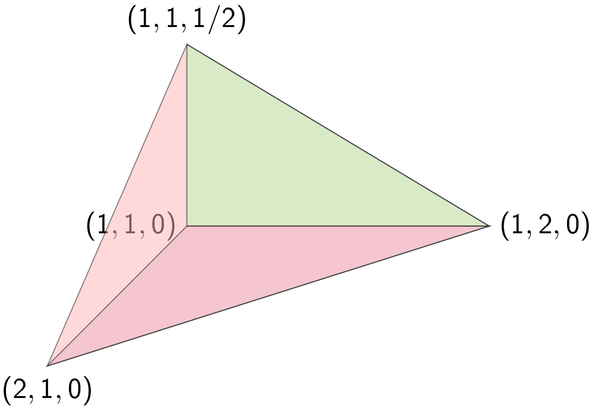
Låt

$$D = \{(x, y, z) : x + y + 2z \leq 3, x \geq 1, y \geq 1, z \geq 0\}.$$

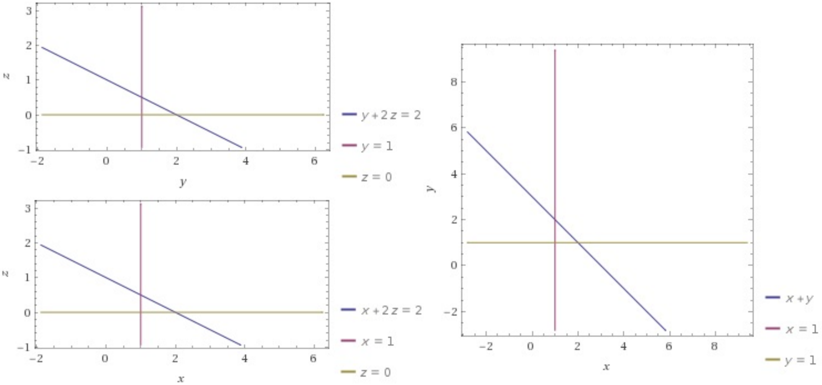
Beräkna

$$\iiint_D x dx dy dz.$$

Bild av D



Tvärsnitt med koordinatplan



Lösning

$$D = \{(x, y, z) : (x, y) \in \widetilde{D}, 0 \leq z \leq (3 - x - y)/2\},$$

där

$$\widetilde{D} = \{(x, y) : x + y \leq 3, x \geq 1, y \geq 1\}$$

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$$\{(x, y) : 1 \leq x \leq 2, 1 \leq y \leq 3 - x\}.$$

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D.v.s.

$$D = \{(x, y, z) : 1 \leq x \leq 2, 1 \leq y \leq 3 - x, 0 \leq z \leq (3 - x - y)/2\}.$$

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$$\ldots = \frac{1}{2} \int_1^2 (2x - 2x^2 + \frac{x^3}{2})dx = \ldots = \frac{5}{48}.$$