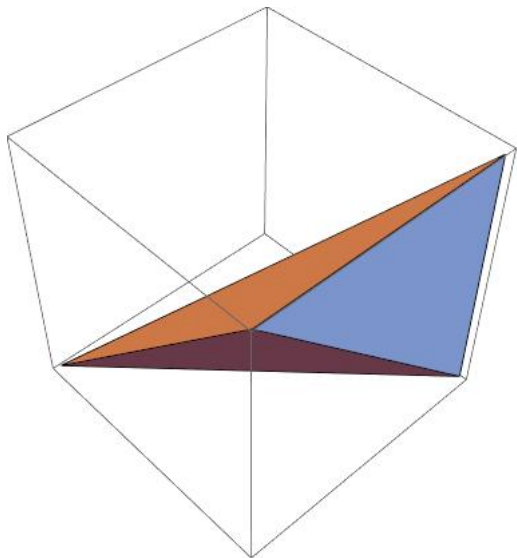


Beräkna

$$\iiint_D (x - y) dx dy dz$$

där D är tetraedern som har hörn i $(0, 0, 0)$, $(1, 1, 1)$, $(1, 1, 0)$ samt $(1, 0, 1)$



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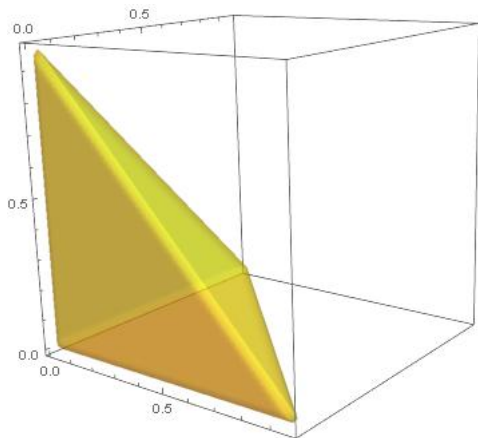
Tetraedern avbildas nu på tetraedern Ω med hörn i $(0, 0, 0)$, $(1, 0, 0)$, $(0, 1, 0)$ och $(0, 0, 1)$ i uvw -rummet.

D.v.s.

$$\Omega = \{(u, v, w) : 0 \leq u \leq 1, 0 \leq v \leq 1 - u, 0 \leq w \leq 1 - u - v\}.$$

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Skalfaktor vid variabelbytet:

$$dxdydz = \left| \frac{d(x, y, z)}{d(u, v, w)} \right| dudvdw$$

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