

Beräkna, om gränsvärdet existerar:

$$\lim_{(x,y) \rightarrow (0,0)} \frac{\cos(x^2 + y^2) - 1}{x^4 + 2x^2y^2 + y^4}.$$

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$$\frac{\cos(x^2 + y^2) - 1}{x^4 + 2x^2y^2 + y^4} = / \cos t = 1 - t^2/2 + b(t)t^4/$$

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$$\frac{1 - (x^2 + y^2)^2/2 + b(x^2 + y^2)(x^2 + y^2)^4 - 1}{x^4 + 2x^2y^2 + y^4}$$

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$$\begin{aligned}
 \frac{\cos(x^2 + y^2) - 1}{x^4 + 2x^2y^2 + y^4} &= / \cos t = 1 - t^2/2 + b(t)t^4 / = \\
 \frac{1 - (x^2 + y^2)^2/2 + b(x^2 + y^2)(x^2 + y^2)^4 - 1}{x^4 + 2x^2y^2 + y^4} &= \\
 \frac{1 - (x^2 + y^2)^2/2 + b(x^2 + y^2)(x^2 + y^2)^4 - 1}{(x^2 + y^2)^2} &= \\
 -\frac{1}{2} + \frac{b(\rho^2)\rho^8}{\rho^4} &
 \end{aligned}$$

$$\begin{aligned}
 \frac{\cos(x^2 + y^2) - 1}{x^4 + 2x^2y^2 + y^4} &= / \cos t = 1 - t^2/2 + b(t)t^4 / = \\
 \frac{1 - (x^2 + y^2)^2/2 + b(x^2 + y^2)(x^2 + y^2)^4 - 1}{x^4 + 2x^2y^2 + y^4} &= \\
 \frac{1 - (x^2 + y^2)^2/2 + b(x^2 + y^2)(x^2 + y^2)^4 - 1}{(x^2 + y^2)^2} &= \\
 -\frac{1}{2} + \frac{b(\rho^2)\rho^8}{\rho^4} &\rightarrow -\frac{1}{2} \text{ då } \rho \rightarrow 0.
 \end{aligned}$$