Exempel

Beräkna, om gränsvärdet existerar:

$$\lim_{(x,y)\to(1,0)} \frac{\sin(x^2 - 2x + y^2 + 1)}{x^2 - 2x + y^2 + 1}.$$

Lösning

$$\lim_{(x,y)\to(1,0)} \frac{\sin(x^2 - 2x + y^2 + 1)}{x^2 - 2x + y^2 + 1}$$

Lösning

$$\lim_{(x,y)\to(1,0)} \frac{\sin(x^2-2x+y^2+1)}{x^2-2x+y^2+1} = /x = 1+t/$$

Lösning

$$\lim_{(x,y)\to(1,0)} \frac{\sin(x^2-2x+y^2+1)}{x^2-2x+y^2+1} = /x = 1+t/ =$$

$$\lim_{(t,y)\to(0,0)} \frac{\sin((1+t)^2-2(1+t)+y^2+1)}{(1+t)^2-2(1+t)+y^2+1}$$

Lösning

$$\lim_{(x,y)\to(1,0)} \frac{\sin(x^2 - 2x + y^2 + 1)}{x^2 - 2x + y^2 + 1} = /x = 1 + t/ =$$

$$\lim_{(t,y)\to(0,0)} \frac{\sin((1+t)^2 - 2(1+t) + y^2 + 1)}{(1+t)^2 - 2(1+t) + y^2 + 1} =$$

$$\lim_{(t,y)\to(0,0)} \frac{\sin(t^2 + y^2)}{t^2 + y^2}$$

Lösning

$$\lim_{(x,y)\to(1,0)} \frac{\sin(x^2-2x+y^2+1)}{x^2-2x+y^2+1} = /x = 1+t/ =$$

$$\lim_{(t,y)\to(0,0)} \frac{\sin((1+t)^2-2(1+t)+y^2+1)}{(1+t)^2-2(1+t)+y^2+1} =$$

$$\lim_{(t,y)\to(0,0)} \frac{\sin(t^2+y^2)}{t^2+y^2} = 1.$$

Lösning

$$\lim_{(x,y)\to(1,0)} \frac{\sin(x^2 - 2x + y^2 + 1)}{x^2 - 2x + y^2 + 1} = /x = 1 + t/ =$$

$$\lim_{(t,y)\to(0,0)} \frac{\sin((1+t)^2 - 2(1+t) + y^2 + 1)}{(1+t)^2 - 2(1+t) + y^2 + 1} =$$

$$\lim_{(t,y)\to(0,0)} \frac{\sin(t^2 + y^2)}{t^2 + y^2} = 1.$$

Ovan använde vi att $t^2 + y^2 \to 0$ då $(t, y) \to (0, 0)$, samt att $\sin s/s \to 1$ då $s \to 0$ med $s = t^2 + y^2$.